

Young Gauger's I N S T R U C T O R,

Being the most Plain and Easy
I N T R O D U C T I O N to that A R T.

Wherein are contained,
The Doctrine of DECIMAL ARITHMETICK;
the Method of Extracting the ROOTS of all
Powers, in a New, Easy, and more Expeditious
Method than hitherto published:

T H E
METHOD of GAUGING all Kinds of VESSELS,
as used by the OFFICERS of EXCISE:

A New, Easy and Short Way of
U L L A G I N G C A S K S by the P E N :

T H E
Description, Construction and Use of the SLIDING
R U L E, with several necessary Amendments; as also
several useful TABLES and PROBLEMS, never before
published.

To which is added,
A N A P P E N D I X,
O F T H E
GAUGING of Irregular Bodies, wherein several Errors of all
former Writers on this Subject are demonstrated and corrected.

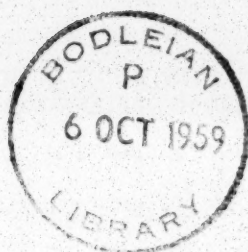
By I S A A C O V E R L E Y,
Late OFFICER of EXCISE.

And likewise,
A S U P P L E M E N T, by the EDITORS:
Wherein is shewn a general Method for finding the Areas, or Solid Con-
tents of all Kinds of curvilinear Figures.

*Tu ne cede malis, sed contra audentior ito,
Quà tua te Fortuna adducit: Vade, Libelle.*

L O N D O N :

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M D C C X L I X.



To the HONOURABLE

CHARLES POLHILL, Esq;
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nion of *Wales*, and Town of
Berwick upon Tweed;

The following T R A C T is most
humbly Dedicated,

By the EDITORS.

E R R A T A.

PAGE 14, *Line 10, read for a Divisor.* P. 15, *l. 32, r. multiply by 6.* P. 265, *l. 2, for Plate IV. r. Plate I. l. 11, for*
 $\frac{x \cdot 2p}{2}$ *r. $\frac{x - 2p}{2}$.* P. 267, *l. 10, f. 3 B. r. 4 B.* P. 268, *l. 9, f.*
 1323 C, *r. 1323 c + 2989 d. l. 18, f. amifs, r. not be amifs.*
 P. 270. *l. 8. for GHGH, r. GH'TH.* P. 273, *l. 23, f. CB*
 $= 15.8$, *r. CB = 10.8.* P. 280, *l. 32, f. a Cone, r. two Cones.*

The Reader is desired to correct these Escapes, before he reads the Book; and any other Blemishes of less Note, which are partly unavoidable in Books of this Sort, it is hoped will be overlook'd with Candour.



T H E

P R E F A C E.



*I*n this Age of Criticism it is a bold Attempt to commence Author, when Volumes of all Arts and Sciences are so copious, and the Tempers and Ideas of Men as different as their Faces.

The Grammarian expects Orthography, the Rhetorician Eloquence, the Logician Dialect, the Mathematician Demonstration; but the Unlearned expect Plainness and Perspicuity.

Among the great Number of Authors on the present Subject, I find few who have adapted their Writings to the last; and those either swoln with a
Redun-

Redundancy of Inutilities, or emaciated for want of Precepts frequently useful.

I doubt not but the experienced Gauger will approve my Alterations of the common Rules of Cask-gauging; for as Mr. Ward observes, in his Young Mathematician's Guide, Let a Cask be supposed, whose Bung Diameter is 31.5, Head Diameter 24.5, and Length 42 Inches; the said Cask being the Middle Zone of a Spheroid, hath the greatest Curvature of any regular Cask, and holds 100.78 Ale Gallons. If a Cask of the same Dimensions shall be a Frustum of a Parabolick Spindle, its Content is 100.01 Gallons of Ale. If the same Cask be supposed the Frustums of two equal Parabolick Conoids abutting on one common Base, the Content thereof will be 93.01 Gallons of Ale. If the same Cask be taken as the Frustums of two Cones abutting upon one common Base, the Content is 92.18 Ale Gallons.

*Hence it appears by the Rules commonly made use of, that the Difference
between*

between the first and second Cask, is not one Gallon; between the third and last, not one Gallon; but between the second and third, is seven Gallons. Hence it is plain, the Gauger is very liable to an Error of seven Gallons.

I have therefore laid down such Rules as divide the Difference between the Spheroidal Cask, and the Conical Cask, into four equal Parts; and consequently the Gauger is liable (at most) only to an Error of 2.15 Gallons in the afore-said Cask.

I have given no Rules in Cask-gauging for a Cask taken as the Frustums of two Cones abutting upon one common Base, as I well know no such Cask was ever made.

I have inserted some Methods of Ul-laging Casks by the Pen, which nearly agree to the Lines of Segments on the Sliding Rule; and also many Problems not before published, of frequent Use in this Art, together with several Tables and their Uses.

I have given the Analytic Investigation of extracting the Roots of all Powers;

ers; and have endeavoured throughout the whole Work to be plain and concise, have used as few Technical Terms as this Work would admit, for which I hope the Learned will pardon me, as I intended it for the Benefit of those, who have not had Grammatical or Mathematical Education.

It was at the Desire of several Officers of the Excise that the following Sheets were published, for whose Service it was more particularly composed; although it may serve as an Enchiridion to all who are concerned in the Business of Gauging.

I shall add no more, but that if this small Tract is found useful to those, for whose Benefit it was compiled, I have my wished for Satisfaction, well remembering the Latin Adage,

Quod ipse Jupiter neque pluens omnibus placet, neque abstinens.

ISAAC OVERLEY.

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T H E

Young Gauger's I N S T R U C T O R.

P A R T I.



IT is absolutely necessary for him, who would be an expert Gauger, to be well skill'd in Arithmetick, which is of two Kinds, *viz.* Whole Numbers and Fractions, and Fractions are either Vulgar or Decimal; but as the latter only is useful in this Art, it must first of all be acquired, and well understood: And this will be attended but with little Difficulty, where the Person already understands Numeration, Addition, Subtraction, Multiplication and Division of Whole Numbers.

B

A

A Decimal Fraction is no more than a Vulgar Fraction confined to certain Denominators, as 10, 100, 1000, 10000, &c. but here the Denominator is not expressed, but is known by a certain Point placed on the left Hand of the Numerator; and this Point always stands in the highest Place of the Denominator. Thus, to express $\frac{2}{10}$, or two Tenths, it will be thus decimally .2; for here the Point stands in the Place of Tens: To express $\frac{5}{100}$, or five Hundredths, it will be .05; here the Point is in the Place of Hundreds: And thus to express $\frac{4}{10}$, $\frac{6}{100}$, $\frac{251}{1000}$, it will be .4, .06, .251: And here observe, in expressing any Fraction decimally, there must always be as many Figures wanting One, as are in the Denominator, which if not given, must be compleated by annexing Cyphers on the left Hand: Thus, it being required to express $\frac{25}{10000}$; since there are five Figures in the Denominator, and but two in the Numerator; wherefore annex Cyphers on the left Hand of the Numerator, to as many Places wanting One, as are in the Denominator, and it becomes .0025, the Decimal required.

This rightly considered, there can arise no Difficulty in the Numeration of Decimals.

But as Decimals are usually denoted by a Point on the left Hand, so on the contrary, Whole Numbers are distinguished by a Point on the right Hand; as thus 4.

But as in this Subject, mixt Numbers do frequently occur, there is no more to be considered, than setting the Point in its proper Place; that is, the Whole Number on the left Hand, and the Decimal Fraction on the Right; as, if it is required to express $26\frac{14}{100}$, it will be 26.14; and the like of any other.

And here it may not be amiss to observe, that as Cyphers placed on the left Hand of a Whole Num-

Number, do not at all increase its Value, so Cyphers placed on the right Hand after a Decimal, do not increase the Value of that Decimal: Thus the Value of .5 and .50 is the same, Five bearing the same Proportion to Ten, as Fifty doth to a Hundred.

Addition of Decimals.

The Operation here is much the same as in Whole Numbers, if due Regard be had in placing the Points of every Number under each other. Thus, let it be required to add together,

27.45, 2.745, 274.5, 2745., .2745

EXAMPLE.

$$\begin{array}{r}
 27.45 \\
 2.745 \\
 274.5 \\
 2745. \\
 .2745 \\
 \hline
 \text{Sum} \quad 3049.9695
 \end{array}$$

This is so plain, it requires no more Examples.

Subtraction of Decimals.

This differs not in its Operation from Whole Numbers, the same Method being observed for placing the Point as in Addition; but if it so happens, that the upper Number hath not so many Places in its Decimal, as the lower Number hath, Cyphers must be annex'd on the right Hand of the upper Number (at least mentally) to make the Number of Places of Decimals equal. The following Examples will render this plain.

B 2

From

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From 274.64 subtract 25.08; from 463.4 subtract 98.72; from 243.004 subtract 124.73.

The Operation stands thus:

From	274.64	463.40	243.004
Subtract	25.08	98.72	124.73
	249.56	364.68	118.274

Multiplication of Decimals.

Multiplication is no more than an Abridgment of Addition, by considering the Multiplicand placed according to the Value of the Figures in the Multiplier, as many Times as the said Figures in the Multiplier express. Thus, were it required to find the Sum of 24 times 576. Here 24 is usually called the Multiplier, and 4 being in the Place of Units, let 576 be written four Times, as below; then 2 being in the Place of Tens, set down twice 576, putting 6, the last Figure of the Multiplicand, in the Place of Tens; the Sum of these Numbers is the Product of 24 by 576.

EXAMPLE.

$$\begin{array}{r}
 576 \\
 576 \\
 576 \\
 576 \\
 576 \\
 576 \\
 576 \\
 \hline
 13824
 \end{array}$$

But this Operation is also much shortned, by a mental Addition of the Multiplicand, placed according to the Value of the Multiplier; and this

this Abridgment is called Multiplication. This is well known to the Learned, and introduced here only as a *Speculum*, in which the Unskilful may see the Investigation of this Rule.

Multiplication of Decimals is perform'd after the same Manner as in Whole Numbers, the Value of the Product being the Thing chiefly to be consider'd; and this Value is known by cutting off as many Decimals in the Product as there are in both Multiplicator and Multiplicand; as in the Examples following.

$$\begin{array}{r}
 \text{Multiplicand} \quad 34.68 \\
 \text{Multiplicator} \quad 23.5 \\
 \hline
 \begin{array}{r}
 17340 \\
 10404 \\
 6936 \\
 \hline
 \end{array} \\
 \text{Product} \quad 814.980
 \end{array}$$

Here are two Decimals in the Multiplicand, and one Decimal in the Multiplicator; wherefore let three Decimals be cut off by a Point in the Product.

But it sometimes happens, there are not so many Figures in the Product, as there are Decimals in the Multiplicator and Multiplicand; in which Case let Cyphers be placed on the left Hand of the Product, until the Number of Figures be equal to the Number of Decimals contained in the two Numbers to be multiplied.

EXAMPLES.

$$\begin{array}{r}
 .07 \\
 .4 \\
 \hline
 .028
 \end{array}
 \qquad
 \begin{array}{r}
 .06 \\
 .05 \\
 \hline
 .0030
 \end{array}
 \qquad
 \begin{array}{r}
 .007 \\
 5. \\
 \hline
 .035
 \end{array}$$

Divi-

Division of Decimals.

Division is an Abridgment of Subtraction, as Multiplication is of Addition, and shews how many Times the Part, or Divisor, may be taken from the Whole or Dividend, and what remains after the last Subtraction.

The Method of performing this, is the same in Decimals, as in Whole Numbers, no other Difficulty arising, than the Knowledge of the Value of the Quotient; which shews how many Times the Divisor is contained in the Number to be divided; and this Value is shewn by cutting off so many Decimals in the Quotient, as the Number of Decimals in the Dividend exceed those of the Divisor.

EXAMPLE.

Divisor Dividend Quotient

23.5) 814.745 (34.67

705

1097

940

1574

1410

1645

1645

(00)

Here the Decimals in the Dividend, exceed those in the Divisor by Two; therefore let two Decimals be cut off in the Quotient.

But

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But if the Decimals in the Dividend and Divisor are equal, the Quotient is a Whole Number.

It sometimes happens, that the Decimals in the Dividend, are not so many as in the Divisor; in these Cases let Cyphers be annex'd on the right Hand of the Dividend at Pleasure; or at least so many as will make the Decimal Places equal in both Numbers: Thus let it be required to divide 2784.6 by 24.34.

$$\begin{array}{r}
 24.34 \overline{) 2784.60000} \quad (114.404 \\
 \underline{2434} \\
 3506 \\
 \underline{2434} \\
 10720 \\
 \underline{9736} \\
 9840 \\
 \underline{9736} \\
 10400 \\
 \underline{9736} \\
 664
 \end{array}$$

Here, by annexing four Cyphers, there are three Decimals more in the Dividend, than there are in the Divisor; therefore let three Decimals be cut off in the Product.

If after the Operation, there are not so many Figures in the Quotient, as are required to be cut off; then the Defect must be supplied by annexing Cyphers on the left Hand of the Quotient.

EXAM-

EXAMPLE.

$$\begin{array}{r}
 957.) \ 27.5406 \ (287 \\
 \underline{1914} \\
 8400 \\
 \underline{7656} \\
 7446 \\
 \underline{6699} \\
 (747)
 \end{array}$$

Here should be four Decimals in the Quotient ; because the Decimals in the Dividend exceed those in the Divisor by Four ; wherefore having but three Places in the Quotient, let one Cypher be annex'd on the left Hand, and it becomes .0287, the Quotient requir'd.

N. B. The Number of Decimals in the Quotient, may always be continued at Pleasure, by annexing Cyphers at the right Hand of the Dividend.

To find a Multiplier, which will perform the Office of a Divisor.

Annex a sufficient Number of Cyphers on the right Hand of an Unit, and divide by the given Divisor ; the Quotient is the Multiplier required.

Let it be required to find a Multiplier to perform the Office of the Divisor 25.

EXAM-

EXAMPLE.

$$\begin{array}{r} 25.) \ 1.00 \text{ (.04 Multiplier required)} \\ \underline{100} \\ \dots \end{array}$$

Therefore, any Number being multiplied by .04, will produce the same Value, as if the same Number was divided by 25.

EXAMPLE.

$\begin{array}{r} 25.) \ 1875. \text{ (75.} \\ \underline{175} \\ 125 \\ \underline{125} \\ \dots \end{array}$	$\begin{array}{r} 1875. \\ \underline{.04} \\ 75.00 \\ \dots \end{array}$
--	---

To reduce a Vulgar Fraction to a Decimal.

Let Cyphers be annexed at Pleasure to the Numerator of the given Fraction, and let it be divided by the Denominator; the Quotient is the Decimal required.

Let it be required to reduce $\frac{7}{8}$ to a Decimal Fraction.

EXAMPLE.

$$8) \ 7.000$$

.875 the Decimal required.

Let it be again required to find the Decimal of a Pound Sterling, equal to 3s 4d, or which is the same $\frac{40}{240}$ or $\frac{4}{24}$.

$$24) 4.000 (.166$$

$$\underline{24}$$

$$160$$

$$\underline{144}$$

$$160$$

$$\underline{144}$$

$$\underline{16}$$

And thus of any other Part of a Pound, Gallon, Hundred, &c.

To find the Value of Decimals in known Parts of Coin, Weight, &c.

This is the Converse of the former, and is thus perform'd: Multiply the given Decimal, by the Number of Species of the highest Denomination contained in the Coin, Weight, &c. of which the Decimal Fraction represents a Part.

Let it be requir'd to find the Value of .28125 of a Pound Sterling.

$$.28125$$

20 Shillings in a Pound.

$$\underline{5.62500}$$

12 Pence in a Shilling.

$$\underline{7.50000}$$

4 Farthings in a Penny.

$$\underline{2.00000}$$

Thus

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Thus the Value of the given Decimal, is five Shillings and seven Pence Halfpenny.

Here observe, the whole Numbers are not to be multiplied. But this Work may be much shortned in finding the known Value of the Decimal of a Pound Sterling, thus:

Observe, how many Times the Number 5 is contained in the two first Figures of the given Decimal; and call the Quotient so many Shillings, and if any Thing remains, annex it to the third Figure, and they will be so many Farthings less one.

EXAMPLE of the Decimal proposed .28125.

In the first two Figures the Number 5 is contained five Times, that is, five Shillings; and the 3 which remains being annexed to the One makes 31, less one leaves 30 Farthings, or seven Pence Halfpenny.

Let the given Decimal be .785 of a Pound Sterling.

OPERATION.

$$\begin{array}{r}
 5) \ 785 \\
 \hline
 15 \text{ remains } 35 \\
 \text{less } 1 \\
 \hline
 4) \ 34 \\
 \hline
 8 \frac{1}{2} \\
 \hline
 \end{array}$$

Thus the Value is fifteen Shillings, and eight Pence Halfpenny.

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Let it be required to find the Value of .28125 of a lb. Weight.

$$\begin{array}{r}
 .28125 \\
 16 \text{ Ounces in a Pound Avoirdupoise.} \\
 \hline
 168750 \\
 28125 \\
 \hline
 4.50000 \\
 16 \text{ Drams in one Ounce.} \\
 \hline
 300000 \\
 50000 \\
 \hline
 8.00000 \\
 \hline
 \hline
 \end{array}$$

Thus the Value is four Ounces, and eight Drams. By these Examples, the Learner, may plainly see the Method of Working in other Qualities, as Gallons, Tuns, Hundreds, &c.

Between two given Numbers to find an Arithmetical mean Proportional.

Half the Sum of the two given Numbers is the Mean Proportional Arithmetical.

What's the Arithmetical Mean between 36 and 64?

$$\begin{array}{r}
 \text{To } 64 \\
 \text{Add } 36 \\
 \hline
 \end{array}$$

$$2) 100$$

The half 50 is the Arithmetical Mean.

Between two given Numbers to find a Mean Proportional Geometrical.

Multiply

Multiply the Numbers by each other, and extract the square Root of the Product.

What's the Geometrical Mean between 9 and 16?

Multiply 16
By 9

$\overline{144}$ (12 The Geometrical Mean
1

22) 44
44
—
0
—

The Method of extracting Roots of all Powers shall be shewn hereafter.

Of EXTRACTION of ROOTS.

To Extract the Square Root.

To extract the square Root, is to discover a Number, which multiplied by itself produces the Number given; and is perform'd in the following Manner.

TABLE of SQUARES.

Roots	1	2	3	4	5	6	7	8	9
Squares	1	4	9	16	25	36	49	64	81

1. Be-

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1. *Beginning at the Right Hand make a Point over the first, third, fifth, &c. Figures.*

2. *From the Table of Squares, or mentally, find the greatest Root contain'd in the first Period on the Left Hand, and set down the Figure as you do a Quotient in Division, setting the Square thereof under the said Period, from which subtract the said Square.*

3. *To the Remainder bring down the next Period of Figures for a Dividend, and double the Root already found for Divisor, and seek how often this Divisor is contain'd in the Dividend, rejecting the Right Hand Figure of the Dividend: This Figure, so found, must be annex'd to the Root last found, and also to the Divisor; this Divisor is to be multiplied by this Figure of the Root last found, and the Product is to be subtracted from the Dividend.*

4. *Bring down the next Period, then double the Part of the Root already found for a second Divisor, and proceed as before.*

The following Examples will render this more plain.

Let it be requir'd to find the Square Root of 181476.

Operation 1st

181476

Operation 2d

181476 (4
16
—
2

Operation

Operation 3d

18¹14⁷6⁶ (42
16

82) 214
164
50

Operation 4th

18¹14⁷6⁶ (426 Root
16

82) 214
164
846) 5076
5076
000

EXPLANATION.

Having first of all divided the given Number into Periods of two Figures, the first Period on the Left Hand is 18, whose nearest Root is 4; which place in the Quotient, and let the Square thereof 16 be subtracted from 18, and there remains 2; to this Remainder bring down the next Period 14, which makes 214; then double the Quotient 4 which is 8; this put in the Place of a Divisor, and seek how often it is contain'd in 21, rejecting the last Figure, viz. 4, which is found to be twice; therefore write 2 in the Quotient, and also annex it to the 8, and it becomes 82 the Divisor, which multiplied by 2 the last Figure found of the Quotient, produces 164; this subtracted from 214 leaves 50; to this bring down the next Period 76, which makes it 5076: Double the Quotient 42, which makes 84; this put in the Place of a Divisor, and enquire how often it is contain'd in 507, rejecting the last Figure 6, and it is found 6 times; this 6 place in the Quotient, and also to the 84 which makes it 846, the second Divisor; this Divisor thus compleated, multiply 6, the last Figure found, the Product is 5076, which being subtracted from 5076 the Dividend, there remains nothing. Thus the Operation is finish'd.

5,

Let

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Let it be requir'd to extract the square Root of 572199960721.

572199960721 (756439 Root
49

1 Divisor 145) 821
725

2 Divisor 1506) 9699
9036

3 Divisor 15124) 66396
60496

4 Divisor 151283) 590007
453849

5 Divisor 1512869) 13615821
13615821

.....

N.B. There are always so many Figures in the Root, as there are Points over the given Number.

What's the square Root of 5817744.

5817744 (2412 Root
4

44) 181
176

481) 577
481

4822) 9644
9644

...

The

The Extraction of the square Root, where there are Decimals, is perform'd in the like Manner; observing the Decimals in the given Number must be an even Number, *viz.* Two, Four, Six, &c. which if not already so, may be compleated by annexing one or more Cyphers, and the Decimals in the Root, when found, will be equal in Number to the Point over the Decimal of the given Number.

What's the square Root of 406.0225?

$$\begin{array}{r}
 \dot{4}0\dot{6}.\dot{0}2\dot{2}5 \text{ (20.15 Root)} \\
 4 \\
 \hline
 401) 00602 \\
 401 \\
 \hline
 4025) 20125 \\
 20125 \\
 \hline
 \dots
 \end{array}$$

Here let two Decimals be cut off in the Root, because there are two Points over the Decimals in the given Number.

A furd Number is that which hath no true Root, and this Case requires an even Number of Cyphers to be annex'd at Pleasure; and the more Cyphers are used in this Case, the nearer the Truth may its Value be approximated.

D

What's

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What's the square Root of 7?

$$\begin{array}{r}
 \dot{7}.00000000 \text{ (2.6457 Root)} \\
 4 \\
 \hline
 46) 300 \\
 \quad 276 \\
 \hline
 524) 2400 \\
 \quad 2096 \\
 \hline
 5285) 30400 \\
 \quad 26425 \\
 \hline
 52907) 397500 \\
 \quad 370349 \\
 \hline
 \quad \quad 27151 \\
 \hline
 \end{array}$$

To Extract the Cube ROOT.

A Cube Root is a Number requir'd to be found, which multiplied by itself, and that Product again by the said Cube Root, will produce the given Number: As if the given Number was 125.

Root	5	
Multiplied by	5	
Produces	25	
Multiplied by	5	
Produces	125	the Cube

To extract the Cube Root, your periodical Points must be over every third Figure, and your Decimals, if there be any, must always be Three, Six, Nine, Five, Twelve,

Twelve, &c. The Number of Figures in the Root, will be equal to the Number of periodical Points, and the Number of Decimals in the Root, to the Number of Points over the Decimal in the given Number.

The common Method of extracting the Cube Root is (as Mr. *Ward* observes in his young Mathematician's Guide) very long and tedious; wherefore I shall (as that Author hath done) reject that Method, and deliver one much more compendious than Mr. *Ward's*; which take as follows.

TABLE of CUBES.

Root	1	2	3	4	5	6	7	8	9	10
Cube	1	8	27	64	125	216	343	512	729	1000

1. Having divided the given Number into Periods of three Figures, that is, setting Points over the First, Fourth, Seventh, Tenth, &c. Figures beginning at the right Hand, by the Table of Cubes, or otherwise, seek the Root of the nearest Cube, either above or below the first Period of the given Number.

2. To this Root thus found, annex as many Cyphers, less one, as there are periodical Points, and multiply this Number thus compleated by 3, and then divide the given Number by this Product.

3. Multiply the Root (completed, as before directed) by itself, and divide the Product by 12.

4. Subtract the Number last produced, from the Number produced by the second Step, and extract the square Root of the Remainder.

5. Add this square Root last found, to half the Root first found and compleated with Cyphers; the Sum is the Cube Root sought.

Let the Cube Root of $14\dot{6}363\dot{1}8\dot{3}$ be required.

Operation 1st.

$\dot{1}4\dot{6}\dot{3}\dot{6}\dot{3}\dot{1}8\dot{3}$ (5 nearest Root to 146.
D₂ Opera-

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Operation 2d.

$$\begin{array}{r}
 500 \text{ suppos'd Root} \\
 \underline{3} \\
 1500) 146363183 \text{ (97575} \\
 \underline{135} \\
 113 \\
 \underline{105} \\
 86 \\
 \underline{75} \\
 113 \\
 \underline{105} \\
 81 \\
 \underline{75} \\
 6
 \end{array}$$

Operation 3d.

$$\begin{array}{r}
 500 \\
 \underline{500} \\
 12) 250000 \\
 \underline{} \\
 20833
 \end{array}$$

Operation 4th.

From 97575 produced by 2d Step
 Subtract 20833 produced by 3d Step

$$\begin{array}{r}
 \dot{7}6\dot{7}4\dot{2} \text{ (277 Root)} \\
 \underline{4} \\
 47) 367 \\
 \underline{329} \\
 547) 3842 \\
 \underline{3829} \\
 13
 \end{array}$$

Operation 5th.

To 277 produced by 4th Step
 Add 250 half the first Root

527 Cube Root required

Let

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Let it be required to extract the Cube Root of 67507824239.

Nearest Root

$ \begin{array}{r} 4000 \\ \underline{3} \\ 12000) \ 67507824(239 \\ \underline{5625652.019} \\ 1333333.333 \\ \underline{4292318.6860} \quad (2071.79 \\ 4 \\ 407) \ 02923 \\ \underline{2849} \\ 4141) \ 7418 \\ \underline{4141} \\ 41427) \ 327768 \\ \underline{289989} \\ 414349) \ 3777960 \\ \underline{3729141} \\ 48819 \end{array} $	$ \begin{array}{r} 4000 \\ \underline{4000} \\ 12) \ 16000000.000 \\ \underline{1333333.333} \\ 2071.79 \text{ Add} \\ \underline{2000.00} \\ 4071.79 \text{ Root required} \end{array} $
--	---

Let the Cube Root of 10 be required.

Nearest Root

$ \begin{array}{r} 2 \\ \underline{3} \\ 6) \ 10.000000(\\ \underline{1.666666} \\ .333333 \\ \underline{1.333333} \quad (1.154 \\ 1 \\ 21) \ 033 \\ \underline{21} \\ 225) \ 1233 \\ \underline{1125} \\ 2304) \ 10833 \\ \underline{9216} \\ 1617 \end{array} $	$ \begin{array}{r} 2 \\ \underline{2} \\ 12) \ 4.000000 \\ \underline{.333333} \\ 1.154 \text{ Add} \\ \underline{1.000} \\ 2.154 \text{ Root required} \end{array} $
---	---

A TABLE of Powers for the more ready finding the supposed Root of any Power.

The

	Root or single Side	Square or 2d Power	Cube or 3d Power	Biquadrate or 4th Power	Surfoldid or 5th Power	Cube squar'd or 6th Power	Second Surfoldid or 7th Power	Biquadrate squar'd or 8th Power	Cube cubed or 9th Power
	Index	Index	Index	Index	Index	Index	Index	Index	Index
	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	
1	1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512	
3	9	27	81	243	729	2187	6561	19683	
4	16	64	256	1024	4096	16384	65536	262144	
5	25	125	625	3125	15625	78125	390625	1953125	
6	36	216	1296	7776	46656	279936	1679616	10077696	
7	49	343	2401	16807	117649	823543	5764801	40353607	
8	64	512	4096	32768	262144	2097152	16777216	134217728	
9	81	729	6561	59049	531441	4782969	43046721	387420489	

The Investigation of the general Method of extracting ROOTS.

Of the Cube Root.

Let the given Number, whose Root is to be extracted be $= b$, and let $a + e = \sqrt[3]{b}$: Then by involving $a + e$ to the third Power we shall have,

$$\begin{array}{r|l}
 1 \div 3a & 1 \left| \begin{array}{l} a^3 + 3a^2e + 3ae^2 + eee = b \\ a^2 + ae + e^2 = \frac{b}{3a} \\ 3 \frac{a^2}{4} + ae + e^2 = \frac{b}{3a} - \frac{aa}{12} \end{array} \right. \\
 2 - \frac{a}{12} & 2 \left| \begin{array}{l} a^2 + ae + e^2 = \frac{b}{3a} - \frac{aa}{12} \\ a + e = \sqrt{\frac{b}{3a} - \frac{aa}{12}} \end{array} \right. \\
 3 \text{ } w & 3 \left| \begin{array}{l} a + e = \sqrt{\frac{b}{3a} - \frac{aa}{12}} \\ a + e = \frac{a}{2} + \sqrt{\frac{b}{3a} - \frac{aa}{12}} = \sqrt[3]{b} \end{array} \right. \\
 4 + \frac{a}{2} & 4 \left| \begin{array}{l} a + e = \frac{a}{2} + \sqrt{\frac{b}{3a} - \frac{aa}{12}} = \sqrt[3]{b} \end{array} \right.
 \end{array}$$

In all the following Operations, let all that Part of the Equation, where e is above the second Power be rejected.

And the Method here proposed, has this Utility, that the Operation is the same, whether you suppose a more or less than just, as will appear thus,

Let $a - e = \sqrt[3]{b}$

$$\begin{array}{r|l}
 1 \div 3a & 1 \left| \begin{array}{l} a^3 - 3a^2e + 3ae^2 - e^3 = b \\ a^2 - ae + e^2 = \frac{b}{3a} \\ 3 \frac{a^2}{4} - ae + e^2 = \frac{b}{3a} - \frac{aa}{12} \end{array} \right. \\
 2 - \frac{aa}{12} & 2 \left| \begin{array}{l} a^2 - ae + e^2 = \frac{b}{3a} - \frac{aa}{12} \\ a - e = \sqrt{\frac{b}{3a} - \frac{aa}{12}} \end{array} \right. \\
 3 \text{ } uu & 3 \left| \begin{array}{l} a - e = \sqrt{\frac{b}{3a} - \frac{aa}{12}} \\ a - e = \frac{a}{2} - \sqrt{\frac{b}{3a} - \frac{aa}{12}} = \sqrt[3]{b} \text{ as before.} \end{array} \right. \\
 4 + \frac{a}{2} & 4 \left| \begin{array}{l} a - e = \frac{a}{2} - \sqrt{\frac{b}{3a} - \frac{aa}{12}} = \sqrt[3]{b} \text{ as before.} \end{array} \right.
 \end{array}$$

Of

Of the ROOT of the fourth Power.

	1	Let $a + e = \sqrt[4]{b}$
1	⊙ 4	2 $a^4 + 4a^3e + 6a^2e^2 = b$
2	÷ $6a^2$	3 $\frac{a^2}{6} + \frac{4ae}{6} + e^2 = \frac{b}{6a^2}$
3	— $\frac{aa}{18}$	4 $\frac{aa}{9} + \frac{4ae}{6} + ee = \frac{b}{6a^2} - \frac{aa}{18}$
4	uu 2	5 $\frac{a}{3} + e = \sqrt{\frac{b}{6a^2} - \frac{aa}{18}}$
5	+ $\frac{2a}{3}$	6 $a + e = \frac{2a}{3} + \sqrt{\frac{b}{6a^2} - \frac{aa}{18}} = \sqrt[4]{b}$

Of the ROOT of the fifth Power.

	1	$a + e = \sqrt[5]{b}$
1	⊙ 5	2 $a^5 + 5a^4e + 10a^3e^2 = b$
2	÷ $10a^3$	3 $\frac{aa}{10} + \frac{ae}{2} + ee = \frac{b}{10a^3}$
3	— $\frac{3}{80}aa$	4 $\frac{aa}{16} + \frac{ae}{2} + ee = \frac{b}{10a^3} - \frac{3}{80}aa$
4	uu 2	5 $\frac{a}{4} + e = \sqrt{\frac{b}{10a^3} - \frac{3aa}{80}}$
5	+ $\frac{3}{4}a$	6 $a + e = \frac{3}{4}a + \sqrt{\frac{b}{10a^3} - \frac{3aa}{80}} = \sqrt[5]{b}$

Of the ROOT of the sixth Power.

	1	$a + e = \sqrt[6]{b}$
1	⊙ 6	2 $a^6 + 6a^5e + 15a^4e^2 = b$
2	÷ $15a^4$	3 $\frac{aa}{15} + \frac{2ae}{5} + ee = \frac{b}{15a^4}$
3	— $\frac{2}{75}aa$	4 $\frac{aa}{25} + \frac{2ae}{5} + ee = \frac{b}{15a^4} - \frac{2}{75}aa$
4	uu 2	5 $\frac{a}{5} + e = \sqrt{\frac{b}{15a^4} - \frac{2}{75}aa}$
5	+ $\frac{4}{5}a$	6 $a + e = \frac{4}{5}a + \sqrt{\frac{b}{15a^4} - \frac{2}{75}aa} = \sqrt[6]{b}$

And

And thus one may proceed to the higher Powers.

But if this Affair be duly consider'd, of comparing the Roots, thus far investigated, a Method of finding Theorems, for extracting the Roots of higher Powers, may be discovered; and that without the Trouble of any other Operation, than those already shewn.

Thus in comparing the four foregoing Theorems, viz.

$$\begin{aligned} \frac{1}{2}a + \sqrt{\frac{b}{3a} - \frac{1}{12}aa} &= \sqrt[3]{b} \\ \frac{2}{3}a + \sqrt{\frac{b}{6aa} - \frac{1}{18}aa} &= \sqrt[4]{b} \\ \frac{3}{4}a + \sqrt{\frac{b}{10a^3} - \frac{3}{80}aa} &= \sqrt[5]{b} \\ \frac{4}{5}a + \sqrt{\frac{b}{15a^4} - \frac{2}{75}aa} &= \sqrt[6]{b} \end{aligned}$$

It may be observed, that $a + \sqrt{\frac{b}{-} - aa}$ is common to each of them; and that the Fractions into which a is multiplied, follow in the Series of $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \&c.$ Whence proceed as follows,

$$\begin{aligned} \frac{1}{2}a + \sqrt{\frac{b}{-} - aa} \\ \frac{2}{3}a + \sqrt{\frac{b}{-} - aa} \\ \frac{3}{4}a + \sqrt{\frac{b}{-} - aa} \\ \frac{4}{5}a + \sqrt{\frac{b}{-} - aa} \\ \frac{5}{6}a + \sqrt{\frac{b}{-} - aa} \\ \frac{6}{7}a + \sqrt{\frac{b}{-} - aa}, \&c. \end{aligned}$$

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It may also be observed, that the Power by which b is divided, is increas'd by the continual Multiplication of a ; and that the Co-efficient of the said Power is increased by the continual Addition of 3, 4, 5, 6, 7, 8, 9, &c. whence arifes,

$$\begin{aligned} \frac{1}{2} a + \sqrt{\frac{b}{3a} - aa} \\ \frac{2}{3} a + \sqrt{\frac{b}{6a^2} - aa} \\ \frac{3}{4} a + \sqrt{\frac{b}{10a^3} - aa} \\ \frac{4}{5} a + \sqrt{\frac{b}{15a^4} - aa} \\ \frac{5}{6} a + \sqrt{\frac{b}{21a^5} - aa} \\ \frac{6}{7} a + \sqrt{\frac{b}{28a^6} - aa}, \&c. \end{aligned}$$

It may further be observed, that the Fraction into which aa is multiplied, is produced by multiplying half the Fraction annex'd to a , into the whole Fraction annex'd to b ; whence arifes,

$$\begin{aligned} \frac{a}{2} + \sqrt{\frac{b}{3a} - \frac{1}{12} aa} \\ \frac{2a}{3} + \sqrt{\frac{b}{6a^2} - \frac{1}{18} aa} \\ \frac{3a}{4} + \sqrt{\frac{b}{10a^3} - \frac{3}{80} aa} \\ \frac{4a}{5} + \sqrt{\frac{b}{15a^4} - \frac{2}{75} aa} \\ \frac{5a}{6} + \sqrt{\frac{b}{21a^5} - \frac{5}{252} aa} \\ \frac{6a}{7} + \sqrt{\frac{b}{28a^6} - \frac{3}{196} aa} \end{aligned}$$

But

But in order to discover a Theorem, whereby the Root of any Power may be extracted.

First, it is to be observed, That the Denominator of the Fraction, into which a is multiplied, is always less by One, than the Index of the given Power; and that the Numerator of the same Fraction is less by One, than the Denominator.

Secondly, That the Index of the Power of a , by which b is divided, is always the same as the Numerator of the aforesaid Fraction.

Thirdly, That the Co-efficient of the said Power of a , is produced by multiplying half the Index of the given Power, into the said Index less by an Unit.

Lastly, That the Fraction, into which aa is multiplied, is the Product of half the Fraction annex'd to a , and the whole Numeral-Fraction annex'd to b .

Let the Theorem of the $\sqrt[30]{b}$ be requir'd.

Then by the first Observation, $29 - 1 = \frac{28}{30 - 1 = 29}$ the Fraction of a .

By the second arises $\frac{b}{a^{28}}$.

By the third $15 \times \frac{b}{a^{28}} = 435$; whence arises $435a^{28}$.

By the last $\frac{14}{29} \times \frac{1}{435} = \frac{14}{12615}$.

The Theorem when compleated is $\frac{28}{29} a + \sqrt[30]{\frac{b}{435a^{28}} - \frac{14}{12615} aa} = \sqrt[30]{b}$.

Let the Theorem of $\sqrt[12]{b}$ be requir'd.

First, $\frac{11}{12} - 1 = \frac{10}{11} a$.

E 2

Second,

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Second, $\frac{b}{10a}$

Third, $6 \times \frac{1}{12-1} = 66$

Fourth, $\frac{5}{11} \times \frac{1}{66} = \frac{5}{726}$

Theorem $\frac{10}{11} a + \sqrt{\frac{b}{66a^{10}} - \frac{5}{726} aa} = \sqrt[11]{b}$

Some Examples of the former THEOREMS.

Let the Cube Root of 14172488 be requir'd.

Having pointed it into Periods of three Figures, it will be 14172488; hence one may perceive the Root to be upwards of 200; then taking 200 equal to a , there arises by the Theorem,

$$100 + \sqrt{\frac{14172488}{600} - \frac{40000}{12}}$$

Which is to be resolv'd as follows.

600) 141724,88	12) 40000
23620.81	3333.33
3333.33	
20287.48	(142.4
1	100 Add.
24) 102	242.4
96	
282) 687	
564	
2844) 12348	
11376	
972	

Here

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Here the Root found is .4 too much; which arises from the suppos'd Root being taken wide; but this suppos'd Root may always be nearly guess'd at thus: The first Period is 14, the Difference between the next greater and next lesser Cube, viz. 8 and 27 is 19, and between 8 and 14 is 6; then say, as 19:10::6:31. Wherefore taking 231 for the suppos'd Root, the true Root 242 would have been produc'd.

Let the Cube Root of 31354496 be requir'd.

Next greater Cube 64

Next less Cube 27

—

37

First Period 31

Next less Cube 27

—

4

As 37 : 100 :: 4 : 10

Add 300

—

Suppos'd Root 310

$$\text{Then } 155 + \sqrt[3]{\frac{31354496}{930} - \frac{96100}{12}} = 316 \text{ Root.}$$

Let the Cube Root of 67.507824239 be requir'd.

Next greater Cube 125

Next less Cube 64

—

61

First

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First Period	67
Next less Cube	64
	3

As 61 : 1000 :: 3 : 049 } is = 4.071783 Root.
 4.000

Suppos'd Root 4.049

Then 2.0245 + $\sqrt{\frac{67.507824239}{12.147} - \frac{16.394401}{12}}$ = Root.

Let the Biquadrate Root of 4857532416 be requir'd.

Next greater	81
Next less	16
	65
First Period	48
Next less	16
	32

As 65 : 100 :: 32 : 50 nearly.

Add	200

Suppos'd Root 250

Then by the Theorem

166.66 + $\sqrt{\frac{4857532416}{375000} - \frac{62500}{18}}$ = 264 Root.

Let the Surfolid Root of 12309502009375 be requir'd. Next

Next greater	3125
Next less	1024
	2101

First Period	1230
Next less	1024
	206

As 2101 : 100 :: 206 : 10 nearly.

Add	400

Suppos'd Root	410
---------------	-----

Then $307.5 + \sqrt{\frac{12309502009375}{689210000} - \frac{50430}{80}} = 415$ Root.

Let the Root be extracted, the sixth Power of which is 145220537353515625.

Next greater	262144
Next less	117649
	144495

First Period	145220
Next less	117649
	27571

As 144 : 100 :: 27 :	15
Add	700

Suppos'd Root	715.
---------------	------

Instead of 715, the suppos'd Root, in order to render the Operation more easy, take 720.

Then

$$\text{Then } 576 + \sqrt{\frac{145220537353515625}{4031078400000} - \frac{1036800}{75}} = 725 \text{ Root.}$$

$$\text{Or } 576 + \sqrt{\frac{1452205373535}{40310784} - \frac{1036880}{75}} = 725$$

From the foregoing THEOREMS, may be deduced the following practical RULES.

I. For the Cube Root.

Divide the given Number, by three Times the suppos'd Root; and from the Quotient subtract one Twelfth of the Square of the suppos'd Root; the square Root of the Remainder, added to half the suppos'd Root, is the true Root requir'd.

II. For the Biquadrate Root.

Divide the given Number, by six Times the Square of the suppos'd Root; and from the Quotient subtract one Eighteenth of the Square of the suppos'd Root; the square Root of the Remainder, added to two Thirds of the suppos'd Root, is the true Root requir'd.

III. For the Sursolid Root.

Divide the given Number, by ten Times the Cube of the suppos'd Root; and from the Quotient subtract three Eightieths of the Square of the suppos'd Root; the square Root of the Remainder, added to three Fourths of the suppos'd Root, is the true Root requir'd.

IV. For the Cube-Squar'd.

Divide the given Number, by fifteen Times the fourth Power of the suppos'd Root; and from the Quotient subtract two Seventy-fifths of the Square of the suppos'd Root; the square Root of the Remainder,

5

added

added to four Fifths of the suppos'd Root, is the true Root requir'd.

Of a SUPERFICIES.

1. A *Superficies*, is a Figure encompassed by one or more Lines; and is either round or angular.
2. A *round Figure*, is that which is contained by one round Line; and is either a *Circle*, or an *Oval*.
3. An *angular Figure*, is that which hath three or more Angles, and takes its Denomination from the Number of its Angles: Thus a Figure of three Angles, is call'd a *Triangle*; a Figure of four Angles, a *Quadrangle*; but Figures of more Angles are usually called *multangular Figures*.
4. A *Triangle*, is a Figure comprehended by three right Lines, of which there are six Kinds; but all require one Method of Gauging.
5. A *Quadrangle*, is a Figure comprehended by four right Lines, and is either a *Parallelogram*, or a *Trapezium*.
6. A *Parallelogram*, is a Figure, whose opposite Sides are equal, and consequently parallel; and is either *Right* or *Oblique*.
7. A *Right-angled Parallelogram*, is that whose Angles are all right; and is either a *Square*, or *Ob-long*.
8. An *Oblique-angled Paralellogram*, is that whose Angles are all *Oblique*; and is either a *Rhombus*, or *Rhomboides*.
9. A *Rhombus*, is an *Oblique Paralellogram*, whose Sides are all equal.
10. A *Rhomboides*, is an *Oblique Paralellogram*, whose opposite Sides are equal, but adjoining Sides unequal.
11. A *Trapezium*, is an irregular Figure of four Sides, whose opposite Sides are not equal.
12. *Multangular Figures*, are either regular or
F irregular;

irregular ; the Regular, are those called *Polygons*, whose Sides as well as Angles are all equal ; and on whose Center a Circle being described, to touch one Angle, will touch all equally.

Irregular Figures are treated of hereafter.

Let us now proceed, to shew how to find the Area, or superficial Content of the Figures before defined ; which are all that usually occur in the Practice of Gauging ; although one may meet with Vessels of a more intricate Form ; but those are best gauged by dividing them into Parts nearly adequate to some of those Forms, and adding the Contents of all those Parts together ; as shall be shewn hereafter.

By the *Area*, (so call'd in the *Gauger's* Phrase) is meant the Quantity of Liquor contained in a Vessel at one Inch deep ; and here it will not be amiss to inform the Reader, that all *Magnitudes* are measured by some *Homogeneous Magnitude* already known : Thus a *Line* is measured by a Linear Inch, Foot, &c. a *Superficies*, by a Square Inch, Foot, &c. and a *Solid*, by a Cubick Inch, Foot, &c.

Wherefore, the Content of a given *Magnitude*, is equal to the Number of *Homogeneous* Inches, Feet, &c. contained therein.

But to return to the present Purpose, *viz.* to determine the Areas of Figures ; in treating of which there is not only shewn the Method of finding their Areas, but some Properties of them are also discovered ; which the Gauger may, perhaps, sometime or other, find useful in his Practice. And first,

Of a CIRCLE.

Having the Diameter, to find the Circumference.

Multiply the given Diameter by 3.1416.

What's

What's the Circumference of a Circle, whose Diameter is 14?

$$\begin{array}{r}
 3.1416 \\
 14 \\
 \hline
 125664 \\
 31416 \\
 \hline
 43.9824 \text{ Circumference.}
 \end{array}$$

Having the Circumference of a Circle, to find the Diameter.

Multiply the given Circumference into .3183.

What's the Diameter of a Circle whose Circumference is 44?

$$\begin{array}{r}
 .3183 \\
 44 \\
 \hline
 12732 \\
 12732 \\
 \hline
 14.0052 \text{ Diameter.}
 \end{array}$$

The Diameter of a Circle being given, to find the Side of a Square equal.

Multiply the Diameter into .886.

Let the given Diameter be 14.

$$\begin{array}{r}
 .886 \\
 14 \\
 \hline
 3544 \\
 886 \\
 \hline
 12.404 \text{ Side of the Square.}
 \end{array}$$

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The Circumference of a Circle being given, to find the Side of a Square equal in Area to the Circle.

Multiply the given Circumference by .282.

Let the given Circumference be 44.

$$\begin{array}{r} .282 \\ 44 \\ \hline 1128 \\ 1128 \\ \hline \end{array}$$

12.408 Side of the Square.

The Diameter of a Circle being given, to find the Side of the greatest Square, which can be inscrib'd in that Circle.

Multiply the given Diameter into .707.

Let the given Diameter be 28.

$$\begin{array}{r} .707 \\ 28 \\ \hline 5656 \\ 1414 \\ \hline \end{array}$$

19.796 Side of inscribed Square.

The Circumference of a Circle being given, to find the Side of the greatest Square that can be inscrib'd.

Multiply the given Circumference into .225.

Let the Circumference given be 88.

$$\begin{array}{r} .225 \\ 88 \\ \hline 1800 \\ 1800 \\ \hline 19.800 \end{array}$$

The

The Diameter of a Circle being given, to find its Area in Inches.

Multiply the Square of the Diameter into .7854.
Let the given Diameter be 14 Inches.

14	.7854	
14	196	
—	—	
56	47124	
14	70686	
—	7854	
Square of Diameter 196	—	
—	153.9384	Content in Inches.

It is necessary here to inform the Reader, that the Area of any Figure in Ale, Wine or Corn Gallons, may be found by dividing its superficial Content in Inches, by the Cubick Inches contain'd in either of those Gallons, *viz.*

The Cubick Inches in the

Ale Gallon	}	are	{	282
Wine Gallon				231
Corn Gallon				268.8

To find the Area of a Circle in Ale or Wine Gallons.

Divide the Square of the Diameter by 359 for Ale, or 294 for Wine.

Let

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Let the Diameter be 40 Inches. (*Fig. 1.*)

40 Multiplied
40 By itself

359) 1600 (4.456 Ale Gallons.
1436

1640
1436

2040
1795

2450
2154

296

But the Learner may perhaps here object against making use of the Divisor 359, seeing the Inches contained in the Ale Gallon, are no more than 282. In order to satisfy him in this Particular, it must be considered, that the Proportion of a Circle, to its circumscribing Square, is as 1 to 1.27324. Now multiplying the Diameter into itself, it produces the Area of the circumscribing Square: Wherefore to reduce this Area, to that of the Circle, it must have a Divisor in Proportion to 282, as 1.27324 is to 1.

EXAMPLE.

As 1, is to 1.27324, so is 282 the Divisor for
282 (Ale Gallons in a Square,

254648
1018592
254648

To 359.05368 The Divisor for Ale Gallons

lons in a Circle, whose Diameter is equal to the Side of the Square.

As 1 is to 1.27324, so is 231 the Divisor for
231 (Wine Gallons in a Square,

$$\begin{array}{r} 127324 \\ 381972 \\ 254648 \\ \hline \end{array}$$

To 294.11844 The Divisor for Wine Gallons in a Circle, whose Diameter is equal to the Side of the Square.

To find the Length, of the Arch Line, of a Segment of a Circle. (Fig. II.)

First, multiply the Line DB (which D must be exactly in the Middle of Arch) by 8 ; and from the Product subtract the Line AB ; the Remainder divide by 3, the Quotient is the Length of the Arch nearly.

$$BD = 10.29$$

8

$$82.32$$

$$AB = 18$$

$$3) 64.32$$

$$21.44 \text{ Arch Line ADB.}$$

To find the Area of a Segment of a Circle in Ale, or Wine Gallons. (Fig. II.)

Square the Line DC, usually called the Versed Sine, and multiply that Square by .4 ; to this Product add the Square of half AB, and extract the square Root of the Sum ; then multiply that Root by DC, and divide the Product by 212 for Ale, or 173 for Wine Gallons.

DC

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$$DC = 5 \quad \text{Half } AB = 9$$

$$\begin{array}{r} 5 \\ \hline \end{array} \quad \begin{array}{r} 9 \\ \hline \end{array}$$

$$\begin{array}{r} 25 \\ \hline \end{array} \quad 81$$

$$\begin{array}{r} .4 \\ \hline \end{array}$$

$$10.0$$

$$81$$

$$\begin{array}{r} 91.0000 \\ \hline \end{array} (9.539$$

$$81$$

$$185) 1000$$

$$925$$

$$1903) 7500$$

$$5709$$

$$19069) 179100$$

$$171621$$

$$(7479)$$

$$9.539 \text{ Root}$$

$$5 = DC$$

$$212) 47.695 \text{ (.224 Area in Ale Gallons.)}$$

$$424$$

$$529$$

$$424$$

$$1055$$

$$848$$

$$207$$

The Segment of a Circle being given, to find the Diameter of that Circle. (Fig. XXIII.)

Multiply half the Chord Line CD by itself, and divide the Product by the Versed Sine EF; to the Quotient thence arising, add the Versed Sine EF, and the Sum will be the Diameter of the Circle.

$$\begin{array}{r}
 \text{Half CD} = 12 \\
 \quad \quad \quad 12 \\
 \quad \quad \quad \hline
 \quad \quad \quad 24 \\
 \quad \quad \quad 12 \\
 \quad \quad \quad \hline
 \text{EF} = 6) 144 \\
 \quad \quad \quad 24 \\
 \quad \quad \quad \hline
 \text{Add } 6 = \text{EF} \\
 \quad \quad \quad \hline
 30 \text{ the Diameter.}
 \end{array}$$

To find the Area of a Sector of a Circle, (Fig. III.) in Ale, or Wine Gallons.

A Sector of a Circle, is a Portion thereof comprehended within two Semi-diameters, and a Part of its Circumference.

Multiply the Semi-diameter (AB) by half the Circumference (BC), and divide the Product by 282 for Ale, or 231 for Wine Gallons.

$$\begin{array}{r}
 \text{AB} = 28 \\
 \text{Half BC} = 22 \\
 \quad \quad \quad 56 \\
 \quad \quad \quad \hline
 \quad \quad \quad 56 \\
 282) 616 \text{ (2.18 Area in Ale Gallons.} \\
 \quad \quad \quad 564 \\
 \quad \quad \quad \hline
 \quad \quad \quad 520 \\
 \quad \quad \quad 282 \\
 \quad \quad \quad \hline
 \quad \quad \quad 2380 \\
 \quad \quad \quad 2256 \\
 \quad \quad \quad \hline
 \quad \quad \quad 124 \\
 \quad \quad \quad \text{G}
 \end{array}$$

Of

Of an ELLIPSIS.

The Transverse and Conjugate Diameters, of an Ellipsis being given; to find the Circumference (Fig. IV.)

Add the square of the Transverse Diameter (AB), to the Square of the Conjugate Diameter (CD), and extract the square Root of that Sum; then double the Root, produced by the Extraction, and add thereto one Third of the Conjugate Diameter, and it will give the Circumference within less than one hundredth Part of the Whole.

AB = 40	CD = 30	50 Root.
40	30	50
-----	-----	-----
1600	900	100 Doubled.
900		10 One Third of CD.
-----		-----
2500	(50 Root	110 Circumference.
25		

...		

To find the Area of an Ellipsis, in Ale, or Wine Gallons.

Multiply the Transverse and Conjugate Diameters together, and divide the Product by 359 for Ale, or 294 for Wine Gallons.

Let the Transverse Diameter be 40 Inches, and Conjugate Diameter 30 Inches.

	40	
	30	

294)	1200.00	(4.08 Area in Wine Gallons.
	1176	

	2400	
	2352	

	48	

Of

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Of a TRIANGLE.

To find the Area of a Triangle, in Ale, or Wine Gallons. (Fig. V.)

First, find the Perpendicular (A D) by applying a String, or strait Ruler, to the Angle (A), and moving it to and fro, till the shortest Distance between the Angle (A), and the Side (B C) be found; that Distance is the Perpendicular.

Multiply this Perpendicular, by half the Base (B C), or half this Perpendicular, by the whole Base (B C), and divide the Product by 282 for Ale, or 231 for Wine Gallons.

$$\text{Half B C} = 87.5$$

$$\text{A D} = 71.3$$

$$\begin{array}{r}
 2625 \\
 875 \\
 \hline
 6125 \\
 282 \overline{) 6238.75} \quad (22.12 \text{ Ale Gallons.} \\
 \underline{564} \\
 598 \\
 \underline{564} \\
 347 \\
 \underline{282} \\
 655 \\
 \underline{564} \\
 91
 \end{array}$$

But it may sometimes happen; that the Perpendicular cannot be conveniently taken: In this Case do thus: *Add all the Sides together, and from half the Sum, subtract each Side separately. Then multiply the three Remainders into one another, and the last Product multiply by half the Sum of the Sides; extract the square Root of this last Number: This Root is to be divided by 282 for Ale, or 231 for Wine Gallons.*

G 2

Ex-

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EXAMPLE.

102	} Sides.	From 200.5	200.5	200.5
124		Subtract 102.	124.	175.
175				
<u> </u>		98.5	76.5	25.5
2) 401		<u>76.5</u>		

200.5 half Sum. 4925
 5910
 6895

7535.25
 25.5

3767625
 3767625
 1507050

192148.875
 200.5

960744375
 38429775000

38525849.4375 (6206.91
 36

122) 252		Area.
244	282) 6206.91 (22.01	
<u> </u>	564	

12406) 85849	
74436	566
<u> </u>	564

124129) 1141343	
1117161	291
<u> </u>	282

1241381) 2418275	
1241381	9
<u> </u>	
1176894	

To

To cut off from a Triangle any Part thereof desired. (Fig. VI.)

Let it be required to cut off a third Part from the Angle C. Divide the Side opposite to the Angle into three equal Parts; then a Line drawn from C, to the first of those Parts, either towards A, or B, as CD, cuts off the third Part required.

But if it be required to cut off a Part from a Point given in one of the Sides, suppose a third Part by a Line drawn from the Point E. Cut off as before, a third Part from the Angle opposite to the Side, on which you would have the trisecting Line to fall, towards that Side on which the Point is given; then from the given Point E, draw an occult Line to D; (the Point which cuts off the third Part of the Triangle) then draw another occult Line parallel to ED, from C to F; then the Line drawn from E to F, cuts off a third Part of the given Triangle.

Of a SQUARE.

The Side of a Square being given, to find the Diagonal; or which is all one, the Diameter of the circumscribing Circle, (Fig. VII.)

Multiply the Side of the Square by 1.414.

Let the Side of the Square given (AB) be 41 Inches.

$$\begin{array}{r} 1.414 \\ 41 \\ \hline 1414 \\ 5656 \\ \hline \end{array}$$

57.974 = Diagonal AD.

The Difference between the Side and Diagonal of a Square being given; to find the Side of the Square.

Multiply

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Multiply the given Difference by 2.414.

Let the given Difference (AC) be 17 Inches.

$$\begin{array}{r}
 2.414 \\
 17 \\
 \hline
 16898 \\
 2414 \\
 \hline
 41.038
 \end{array}$$

The Side of a Square being given, to find its Area in Ale, or Wine Gallons.

Multiply the Side by itself, and divide the Product by 282 for Ale, or 231 for Wine Gallons.

Given Side AB = 41

$$\begin{array}{r}
 41 \\
 \hline
 41 \\
 164 \\
 \hline
 282) 1681 \text{ (5.96 Area in Ale Gallons.)} \\
 1410 \\
 \hline
 2710 \\
 2538 \\
 \hline
 1720 \\
 1692 \\
 \hline
 28
 \end{array}$$

Of an OBLONG.

The Sides of an Oblong being given, to find the Diagonal. (Fig. VIII.)

Add

Add the Square of the longest Side AB, to the Square of the shortest Side AC; the square Root of the Sum, is equal to the Diagonal AD.

Longest Side AB = 201

$$\begin{array}{r} 201 \\ \hline 201 \\ \hline 4020 \\ \hline 40401 \end{array}$$

Shortest Side AC = 82

$$\begin{array}{r} 82 \\ \hline 164 \\ \hline 656 \\ \hline 6724 \end{array} \left. \begin{array}{l} \\ \\ \\ \end{array} \right\} \text{Add.}$$

$$\begin{array}{r} 40401 \\ \hline 47125 \end{array} \quad (217 = \text{Diagonal AD.})$$

$$\begin{array}{r} 4 \\ \hline 41 \overline{) 71} \\ \underline{41} \\ 427 \overline{) 3025} \\ \underline{2989} \\ 36 \end{array}$$

The Sides of an Oblong being given, to find the Area, in Beer, or Wine Gallons.

Multiply

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Multiply the longest Side, by the Shortest; and divide the Product by 282 for Beer, or 231 for Wine Gallons.

Longest Side 201
Shortest Side 82

402
1608

282) 16482 (58.44 Area in Ale Gallons.
1410

2382
2256

1260
1128

1320
1128

192

Of a RHOMBUS.

To find the Area of a Rhombus, in Ale, or Wine Gallons. (Fig. IX.)

Find the Perpendicular AC, by the Directions given in the Triangle; then multiply the Side of the Rhombus, by this Perpendicular, and divide the Product by 282 for Ale, or 231 for Wine Gallons.

Side

Side AB = 121.

Perp. AC = 109.

$$\begin{array}{r}
 1089 \\
 1210 \\
 282 \overline{) 13189} \quad (46.76 \quad \text{Area in Ale Gallons.} \\
 \underline{1128} \\
 1909 \\
 \underline{1692} \\
 2170 \\
 \underline{1374} \\
 1960 \\
 \underline{1692} \\
 268
 \end{array}$$

Of a RHOMBOIDES.

To find the Area of a Rhomboides, in Ale, or Wine Gallons. (Fig. X.)

Multiply the longest Side AB, by the Perpendicular CI, found as before; and divide the Product by 282 for Ale, or 231 for Wine Gallons.

AB = 185

CD = 92

$$\begin{array}{r}
 370 \\
 1665 \\
 231 \overline{) 17020} \quad (73.67 \quad \text{Area in Wine Gallons.} \\
 \underline{1617} \\
 850 \\
 \underline{693} \\
 1570 \\
 \underline{1386} \\
 1840 \\
 \underline{1617} \\
 223
 \end{array}$$

H

Of

Of a TRAPEZIUM.

To find the Area of a Trapezium, in Ale, or Wine Gallons. (Fig. XI.)

Fix with a Nail, or otherwise, a chalk'd Line, in the most acute Angle A, and carrying the other End, to the opposite Angle, strike the Line AC; this Line is call'd the Base: To this Base, take two Perpendiculars, from the other Angles, as BF and DE; the Sum of these Perpendiculars multiplied by half the Base AC, and the Product divided by 282 gives Ale, or by 231 gives Wine Gallons.

$$\begin{array}{r}
 \text{BF} = 84 \\
 \text{DE} = 63 \\
 \hline
 \text{Half AC} = 147 \\
 \hline
 \begin{array}{r}
 147 \\
 441 \\
 \hline
 1470
 \end{array} \\
 \hline
 282 \overline{) 15141} \quad (53.69 \text{ Area in Ale Gallons.} \\
 \underline{1410} \\
 1041 \\
 \underline{846} \\
 1950 \\
 \underline{1692} \\
 2580 \\
 \underline{2538} \\
 42
 \end{array}$$

Of

Of Regular POLYGONS.

Regular Polygons, have their Names from the Number of the Sides, or Angles, which are equal: Thus a regular Polygon of five Sides, (Fig. XII.) is call'd a Pentagon; of six Sides, (Fig. XIII.) a Hexagon; of seven Sides, (Fig. XIV.) a Heptagon, &c.

The Method of finding the Areas of all regular Polygons, is the same, viz. First of all, find the Center of the Polygon: Thus, if the Polygon be of an even Number of Sides, draw a Line from any one Angle to its opposite Angle, the Middle of that Line is the Center of the Polygon.

But if the Number of Sides be not even; then draw two Lines from any of the Angles, to the Middle of their opposite Sides, and where these Lines intersect each other, is the Center.

From this Center, measure a Perpendicular to the Middle of one of the Sides, multiply this Perpendicular into half the Sum of the Sides, or the whole Sum of the Sides into half the Perpendicular, and divide the Product by 282 for Ale, or 231 for Wine Gallons.

$$\begin{array}{r}
 \text{(Fig. XII.) } 50 = \text{one Side} \\
 \quad \quad \quad 5 = \text{Number of Sides.} \\
 \quad \quad \quad \hline
 \quad \quad \quad 250 \\
 \quad \quad \quad 17.2 = \text{Half Perpendicular.} \\
 \quad \quad \quad \hline
 \quad \quad \quad 500 \\
 \quad \quad \quad 1750 \\
 \quad \quad \quad \hline
 \quad \quad \quad 250 \\
 \quad \quad \quad \hline
 282) 4300.0 \text{ (15.2 Area in Ale Gallons.} \\
 \quad \quad \quad 282 \\
 \quad \quad \quad \hline
 \quad \quad \quad 1480 \\
 \quad \quad \quad 1410 \\
 \quad \quad \quad \hline
 \quad \quad \quad 700 \\
 \quad \quad \quad 564 \\
 \quad \quad \quad \hline
 \quad \quad \quad 136 \\
 \quad \quad \quad \hline
 \quad \quad \quad 136 \\
 \quad \quad \quad \hline
 \quad \quad \quad 0
 \end{array}$$

H 2

Fig.

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(Fig. XIII.) $39.7 =$ one Side.
 $6 =$ Number of Sides.

$$\begin{array}{r} 238.2 \\ 17.2 \text{ Half Perpendicular.} \\ \hline 4764 \\ 16674 \\ 2382 \\ \hline \end{array}$$

231) 4097.04 (17.73 Wine Gallons.
 231

$$\begin{array}{r} 1787 \\ 1617 \\ \hline 1700 \\ 1617 \\ \hline 834 \\ 693 \\ \hline 141 \end{array}$$

(Fig. XIV.) $60 =$ one Side.
 $7 =$ Number of Sides.

$$\begin{array}{r} 2) 420 \\ 210 = \text{Half Sum of Sides.} \\ 62.2 = \text{Perpendicular.} \\ \hline 420 \\ 420 \\ 1260 \\ 282) 130620 (46.3 \text{ A.G.} \\ 1128 \\ \hline 1782 \\ 1692 \\ \hline 900 \\ 846 \\ \hline 54 \end{array}$$

The

The Side of any regular Polygon being given, to find the Area in Ale, or Wine Gallons.

Multiply the Square of the Side, of the given Polygon, by the Tabular Number corresponding to the Number of Sides, of the said Polygon; the Product is the Area, in Ale, or Wine Gallons.

Number of Sides.	Ale.	Wine.
Five	.006101	.007447
Six	.009212	.011245
Seven	.012888	.015733
Eight	.017121	.020899
Nine	.021921	.026760
Ten	.027280	.033302
Eleven	.033210	.040541
Twelve	.039700	.048464

Let the Area of the five-sided Polygon (Fig. XII.) be required, in Ale and Wine Gallons, whose Side is 50 Inches.

$$\begin{array}{r} 50 = \text{Side.} \\ 50 \\ \hline \end{array}$$

$$2500 = \text{Square of the Side.}$$

Tabular Number	.006101	.007447
	2500	2500
	03050500	03723500
	012202	014894
A. G. =	15.252500	W. G. = 18.617500.

Qf

Of Multangular Figures.

The Content, of any straight lined multangular Figure, may be found, by dividing them into several Trapezia, or Triangles, as may be found most convenient: The Contents of which being added together, will give that of the Figure proposed: Thus in *Fig. XV.* by drawing the right Line BC, the proposed Figure, is divided into the Trapezium BCDE, and the Triangle ABC; the Contents of which may be found as follows.

BD 266
Half Sum 20 of Perpendiculars.

1320 = Content of Trapezium.

1120 = Content of Triangle.

282) 2440 (8.65 Area in Ale Gallons.

2256

1840

1692

1480

1410

70

CG = 11

EH = 29

AB = 56

Half FC = 20

2) 40

20

1120 Content of Triangle.

Now, all the foregoing Figures, are suppos'd to be the Bottoms of Vessels, whose Sides are perpendicular thereto; therefore when the Area is found,

found, the Quantity of Liquor in the Vessel is found, by multiplying the Area of the Bottom, by the Depth of the Liquor.

But as Vessels are seldom made with that geometrical Accuracy, which this Art requires; it will be necessary to lay down the Methods us'd in Practice.

The Practical Method, is (when Vessels are fix'd) to tabulate their Contents: Such as Brewers Tuns, Brewers Coolers, Distillers Backs, &c. But in those Vessels, which are us'd for working Liquors, the wet Inches are not easily attainable; therefore those Vessels are tabulated backward, as will more plainly appear, by the following Examples.

Here, first of all, is to be consider'd the best Method of taking Dimensions, which in Figures approaching nearly to Oblongs, is to take the Length in several Places, as seem most convenient; then add all these Lengths together, and divide the Sum by the Number of Dimensions; and the Quotient will be the mean Length. The same Method is to be us'd for the mean Breadth.

But in this Case, ten Dimensions are usually sufficient for a mean Length, and as many for a mean Breadth, which are most commodiously taken in this Manner.

Let there be a Brewer's working Tun, whose Top, or Bottom, is represented by *Fig. XVI*; here the Lines AB, CD, EF, GH, represent the Dimensions taken at the Top, and Bottom, which are eight in Number; the other two Dimensions of the Length, are to be taken close to the Sides AB and GH, at about half the Tun's Depth. The Lines AG, IK, LM, BH, represent the Breadths taken at Top, and Bottom; the other two Breadths are taken close to the Sides AG and BH, at about half the Tun's Depth.

And

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And here it is necessary to observe, that if there be any significant Disparity, in the Diagonals of the Tun, it is best gaug'd as a Trapezium.

Now, suppose the Lengths, and Breadths, taken as before, to be as follows.

Lengths.	Breadths.
120. 4	75. 1
120. 2	75. 4
120. 6	74. 9
120. 3	75. 2
120. 5	75. 4
120. 7	75. 1
120. 1	75. 2
120. 4	74. 9
120. 3	75. 3
120. 6	75. 2

Mean Length 120.4 1. Mean Breadth 75.1 7.

Here the Division, by the Number of Dimensions, (*viz.* 10) is saved by cutting off a Decimal more in the Sum, than is in either of the Dimensions: And in the following Operation, the second Place of Decimals may be rejected.

$$\begin{array}{r}
 120.4 = \text{Length.} \\
 75.1 = \text{Breadth.} \\
 \hline
 1204 \\
 6020 \\
 8428 \\
 282) 9042.04 \quad (32.06 \text{ Area in Ale Gallons.} \\
 \underline{846} \qquad \qquad 10 \\
 \hline
 582 \qquad 320.6 \text{ Content in Ale Gal-} \\
 \underline{564} \qquad \text{lons at ten Inches deep.} \\
 1804 \\
 \underline{1692} \\
 112
 \end{array}$$

Our

Our next Business is to form the Decade of this Tun; which may be done thus:

At the most convenient dipping Place of the Tun, cut a large Notch on the Side thereof, and at that Place measure the Depth of the Tun, suppose 40 Inches; then let so many Gallons of Liquor be poured into the Tun, as may cover the Bottom, or rather more, suppose 16 Gallons; and at the dipping Place aforesaid, take the Depth of Liquor, suppose .5; this being subtracted from 40 Inches, the Depth of the Tun, leaves the neat Depth 39.5; which multiplied into the Tun's Area, and the Product added to the Fall of the Tun, (16 Gallons) will give the whole Content of the Tun in Gallons.

$$\begin{array}{r}
 32.06 \text{ Area.} \\
 39.5 \text{ Neat Depth.} \\
 \hline
 16030 \\
 28854 \\
 9618 \\
 \hline
 1266.370 \\
 16.000 \text{ Add.} \\
 \hline
 1282.370 \text{ Whole Content in Ale Gall.}
 \end{array}$$

Now, whereas 32 Gallons are contain'd in a Barrel of Ale, and 36 Gallons in a Barrel of Beer; the whole Content is to be divided by 32 to find the Quantity of Ale Barrels, or by 36 to find the Quantity of Beer Barrels, in the whole Content.

EXAMPLE.

$$\begin{array}{r}
 32) 1282 \text{ (40} \\
 \quad 128 \\
 \hline
 \text{Remains } 02 \text{ Ale Gallons.} \\
 \quad \quad 1
 \end{array}$$

36

$$\begin{array}{r}
 36) 1282 \text{ (35} \\
 \underline{108} \\
 202 \\
 \underline{180} \\
 \text{---}
 \end{array}$$

Remains 22 Gallons or 2 Firkins, 4 Gallons.

Therefore the whole Content of this Tun in Ale Measure is 40 Barrels, and 2.37 Gallons; and in Beer Measure is 35 Barrels, 2 Firkins, 4.37 Gallons.

The DECADE.

	Area in Gall.	Cont. in B. Barrels.			Cont. in Ale Bar.		
		B.	F.	G.	B.	F.	G.
Whole Depth 40.0	32.06	35	2	4.37	40	0	2.37
Fall 5	---	Cont. of Fall			Cont. at ten Inches.		
Neat Depth 39.5	---	B.	F.	G.	B.	F.	G.
		0	1	7	8	3	5.6
					10	0	0.6

Now to Tabulate this Tun, in order that it may be gauged by the dry Inches. Subtract the Area of the Tun in Barrels, Firkins and Gallons, from the whole Content, and again from the Remainder; and so continually, until you arrive at the Fall of the Tun: Here the Area of the Tun in Beer is 3 Firkins 5.06 Gallons; and in Ale is 1 Barrel and .06 of a Gallon; which, by a continued Subtraction will form the following Tables.

A BEER TABLE.

Inches	B.	F.	G.	Inches	B.	F.	G.
Full	35	2	4.37	20	17	3	2.17
		3	5.06			3	5.06
1	34	2	8.31	21	16	3	6.11
		3	5.06			3	5.06
2	33	3	3.25	22	16	0	1.05
		3	5.06			3	5.06
3	32	3	7.19	23	15	0	4.99
		3	5.06			3	5.06
4	32	0	2.13	24	14	0	8.93
		3	5.06			3	5.06
5	31	0	6.07	25	13	1	3.87
		3	5.06			3	5.06
6	30	1	1.01	26	12	1	7.81
		3	5.06			3	5.06
7	29	1	4.95	27	11	2	2.75
		3	5.06			3	5.06
8	28	1	8.89	28	10	2	6.69
		3	5.06			3	5.06
9	27	2	3.83	29	9	3	1.63
		3	5.06			3	5.06
10	26	2	7.77	30	8	3	5.57
		3	5.06			3	5.06
11	25	3	2.71	31	8	0	0.51
		3	5.06			3	5.06
12	24	3	6.65	32	7	0	4.45
		3	5.06			3	5.06
13	24	0	1.59	33	6	0	8.39
		3	5.06			3	5.06
14	23	0	5.53	34	5	1	3.33
		3	5.06			3	5.06
15	22	1	0.47	35	4	1	7.27
		3	5.06			3	5.06
16	21	1	4.41	36	3	2	2.21
		3	5.06			3	5.06
17	20	1	8.35	37	2	2	6.15
		3	5.06			3	5.06
18	19	2	3.29	38	1	3	1.09
		3	5.06			3	5.06
19	18	2	7.23	39	0	3	5.03
		3	5.06				

The Table thus compleated to 39 Inches, *examine it in this Manner.* Here remains .5 of the neat Depth, which multiplied into 32.06 Gallons

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(the Area) produces 16.03 Gallons, to which add 16 Gallons (the Fall of the Tun) the Sum is 32.03 Gallons, or 3 Firkins and 5.03 Gallons, which is the same as is found in the Table at 39 Inches.

In tabulating a Tun, much Labour is sav'd, if instead of writing the Area, at every Inch, there be set down on a small Piece of Paper the said Area; and move it downwards as you subtract; then the Table will appear fair.

Let us now tabulate this Tun in Ale, whose
B. F. G.
Area is 1 0 2.37.

ALE GALLONS.

Inches	B.	F.	G.	Inches	B.	F.	G.
Full.	40	0	2.37	20	20	0	1.17
1	39	0	2.31	21	19	0	1.11
2	38	0	2.25	22	18	0	1.05
3	37	0	2.19	23	17	0	0.99
4	36	0	2.13	24	16	0	0.93
5	35	0	2.07	25	15	0	0.87
6	34	0	2.01	26	14	0	0.81
7	33	0	1.95	27	13	0	0.75
8	32	0	1.89	28	12	0	0.69
9	31	0	1.83	29	11	0	0.63
10	30	0	1.78	30	10	0	0.57
11	29	0	1.71	31	9	0	0.51
12	28	0	1.65	32	8	0	0.45
13	27	0	1.59	33	7	0	0.39
14	26	0	1.53	34	6	0	0.33
15	25	0	1.47	35	5	0	0.27
16	24	0	1.41	36	4	0	0.21
17	23	0	1.35	37	3	0	0.15
18	22	0	1.29	38	2	0	0.09
19	21	0	1.23	39	1	0	0.03

The

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The Examination of the Ale Table, is the same as of the Beer; 32.03 Gallons being in Ale Measure, 1 B, 0 F, 0.03 G, or one Barrel, and three hundredth Parts of a Gallon.

However, as an Error may sometimes happen at the Beginning of a Table, which if not timely corrected, will cause an Error throughout; the Operation will be more certain if it be examined at every ten Inches, by subtracting the Content at ten Inches continually: Thus,

B E E R.			A L E		
B.	F.	G.	B.	F.	G.
35	2	4.37	40	0	2.37
8	3	5.6	10	0	0.6
26	2	7.77	30	0	1.77 at 10 Inches.
8	3	5.6	10	0	0.6
17	3	2.17	20	0	1.17 at 20 Inches.
8	3	5.6	10	0	0.6
8	3	5.57	10	0	0.57 at 30 Inches.

To Gauge and Tabulate a Brewer's Cooler.

Let us suppose (*Fig. 16.*) to be a Cooler, whose Dimensions are as follow.

AB 94.2	AG 50.2
CD 94.5	IK 49.8
EF 93.9	LM 49.9
GH 94.4	BH 50.0
4) 377.0	4) 199.9
94.25 Mean Length.	49.975 Mean Breadth.

94.25

94.25 Length.
49.97 Breadth.

65975
84825
84825
37700

282) 4709.6725 (16.70 Area in Ale Gallons.
282

1889
1692

1976
1974

27

Whereas *Brewers Worts*, having lain some Time in the Cooler, have a considerable Settling, it is usual to allow one tenth Part; which must be taken from the Area before it be tabulated: Thus,

16.70 Area
1.67 Subtract.

10) 15.03 Neat Area at one Inch.

1.503 Neat Area at one Tenth of an Inch.

The Cooler being gaug'd by the wet Inches, it is to be tabulated forward by the continual Addition of its Area, on one Tenth of an Inch, *viz.* 1.5 Gallons; the other Decimal Places being inconsiderable in this Case.

Beer

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Beer Table. Ale Table.			Beer Table. Ale Table.		
Inch.	B. F. G.	B. F. G.	Inch.	B. F. G.	B. F. G.
.1	0 0 1.5	0 0 1.5	3.1	1 1 1.5	1 1 6.5
.2	0 0 3.0	0 0 3.0	3.2	1 1 3.0	1 2 0.0
.3	0 0 4.5	0 0 4.5	3.3	1 1 4.5	1 2 1.5
.4	0 0 6.0	0 0 6.0	3.4	1 1 6.0	1 2 3.0
.5	0 0 7.5	0 0 7.5	3.5	1 1 7.5	1 2 4.5
.6	0 1 0.0	0 1 1.0	3.6	1 2 0.0	1 2 6.0
.7	0 1 1.5	0 1 2.5	3.7	1 2 1.5	1 2 7.5
.8	0 1 3.0	0 1 4.0	3.8	1 2 3.0	1 3 1.0
.9	0 1 4.5	0 1 5.5	3.9	1 2 4.5	1 3 2.5
1.0	0 1 6.0	0 1 7.0	4.0	1 2 6.0	1 3 4.0
1.1	0 1 7.5	0 2 0.5	4.1	1 2 7.5	1 3 5.5
1.2	0 2 0.0	0 2 2.0	4.2	1 3 0.0	1 3 7.0
1.3	0 2 1.5	0 2 3.5	4.3	1 3 1.5	2 0 0.5
1.4	0 2 3.0	0 2 5.0	4.4	1 3 3.0	2 0 2.0
1.5	0 2 4.5	0 2 6.5	4.5	1 3 4.5	2 0 2.5
1.6	0 2 6.0	0 3 0.0	4.6	1 3 6.0	2 0 5.0
1.7	0 2 7.5	0 3 1.5	4.7	1 3 7.5	2 0 6.5
1.8	0 3 0.0	0 3 3.0	4.8	2 0 0.0	2 1 0.0
1.9	0 3 1.5	0 3 4.5	4.9	2 0 1.5	2 1 1.5
2.0	0 3 3.0	0 3 6.0	5.0	2 0 3.0	2 1 3.0
2.1	0 3 4.5	0 3 7.5	5.1	2 0 4.5	2 1 4.5
2.2	0 3 6.0	1 0 1.0	5.2	2 0 6.0	2 1 6.0
2.3	0 3 7.5	1 0 2.5	5.3	2 0 7.5	2 1 7.5
2.4	1 0 0.0	1 0 4.0	5.4	2 1 0.0	2 2 1.0
2.5	1 0 1.5	1 0 5.5	5.5	2 1 1.5	2 2 2.5
2.6	1 0 3.0	1 0 7.0	5.6	2 1 3.0	2 2 4.0
2.7	1 0 4.5	1 1 0.5	5.7	2 1 4.5	2 2 5.5
2.8	1 0 6.0	1 1 2.0	5.8	2 1 6.0	2 2 7.0
2.9	1 0 7.5	1 1 3.5	5.9	2 1 7.5	2 3 0.5
3.0	1 1 0.0	1 1 5.0	6.0	2 2 0.0	2 3 2.0

Now

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Now let a dipping Place be chosen most convenient to yourself, and there cut a Notch, or set any other immoveable Mark; this Cooler being charg'd, dip it at this fixed Place; supposing it to dip 4.9 Inches; then let this Wort down into the Tun, and dip it there, and supposing it dips 38 dry Inches, opposite to which in the Beer Table of the Tun, is 1 Barrel, 3 Firkins, 1 Gallon. Now seek in the Beer Table of the Back for the nearest Content to this, and against 4.3 Inches, there will be found 1 Barrel, 3 Firkins, 1.5 Gallons. Therefore the fixed dipping Place is discovered to dip .6 too much. Consequently, whenever any one dips at this Place, .6 must necessarily be subtracted from that Dip: And this as a Memorandum is to be put under the dipping Place in this Manner, —0.6: But if the same was to be added, it must be marked thus + 0.6.

In Victuallers Backs, the tenth Part of the Area is not subtracted; and as they are small, the Depth in the Middle is sufficiently near; which being taken, it is to be compared with your dipping Place: Thus, suppose the Depth in the Middle, to be 3.8 Inches, and at the dipping Place, 2.6 Inches; the Difference 1.2 is always to be added, and is to be mark'd on the Cooler, thus + 1.2.

To Gauge a Cylinder, (Fig. XVII.)

Multiply the Area of the Base, by the Depth.

Let the Diameter AB = 28.8 Inches; then the Area will be 2.31 Ale Gallons.

$$\begin{array}{rcl} \text{Area} & = & 2.31 \\ \text{Depth CD} & = & 30 \text{ Inches.} \\ \hline \end{array}$$

69.30 Content in Ale Gallons.

To

To Gauge a Cone. (Fig. XVIII.)

Multiply the Area of the Base, by one Third of the Attitude.

Let the Diameter AB be 28.8 Inches, the Depth CD = 30 Inches, as in the last Example.

$$\begin{array}{r} 2.31 \text{ Area in Ale Gallons.} \\ 10 \text{ one 3d of the Attitude.} \\ \hline 23.10 \text{ Content in Ale Gallons.} \end{array}$$

To Gauge a Pyramid. (Fig. XIX.)

Cones and *Pyramids* require the same Method of Gauging, viz. multiply the Area of the Base, by one Third of the Attitude.

Let Fig. XIX. represent a square Pyramid, the Side of whose Base is 16 Inches, and Attitude 34 Inches.

$$\begin{array}{r} 16 \text{ Side of the Base.} \\ 16 \\ \hline 96 \\ 16 \\ \hline 282) 256.000 \text{ (.907 Area in Ale Gallons.} \\ 2538 \\ \hline 2200 \\ 1974 \\ \hline 226 \end{array}$$

$$\begin{array}{r} .907 \text{ Area.} \\ 11.3 \text{ one Third of Depth.} \\ \hline 2721 \\ 907 \\ 907 \\ \hline 10.2491 \text{ Content.} \end{array}$$

K

But

But it sometimes happens, as in this last Example, that the third Part of the Altitude, or Depth, cannot be truly taken: In this Case the shortest, and most exact Way is, to multiply the Area by the whole Length, and divide the Product as in the following Operation.

$$\begin{array}{r}
 .907 \text{ Area.} \\
 34 \text{ Whole Length.} \\
 \hline
 3628 \\
 2721 \\
 \hline
 3) 30.838 \\
 \hline
 10.279 \text{ Content.}
 \end{array}$$

It may not be improper here to define a Pyramid, lest the Learner may not be already acquainted therewith. *A Pyramid is a Figure whose Base may be in any Form, whose upper End terminates in a Vertex, and all right Lines drawn from the Edge of the Base, to the Vertex, will equally touch the Side of the Pyramid. Hence it appears a Cone is a Pyramid, and requires the same Method of Gauging.*

Of *Pyramids* some are right; that is, when a Perpendicular being let fall from the Vertex to the Base, passes exactly through the Center of the Base; (as *Fig. 19.*) Others are oblique, that is, when a Perpendicular being let fall from the Vertex, passes not through the Center of the Base; (as *Fig. 20.*) But the Method of Gauging is the same in each of them; the Perpendicular CD being always the Altitude of the Pyramid.

To Gauge the Frustum of a Cone, or Pyramid.

There are as many different Kinds of Pyramids, as there are different plane Superficies; and the Frustums of each of them may be gaug'd in a different Manner: But to lay down many Rules would appear tedious to the Learner to practice, seeing one general Rule may be delivered that will answer the Purpose. Let the Area of the Bottom be multiplied by the Area of the Top, and the square Root of the Product extracted; then to this Root so found, let the Areas of the Bottom and Top be added, and this Sum multiplied by one Third of the Length, will produce the Content.

Let (*Fig. XXI.*) be the Frustum of a Cone, standing upon the lesser Base.

Diameter A B = 26.8 the Area in Ale Gallons is 2.00.

Diameter C D = 32.9 the Area in Ale Gallons is 3.01.

Mean Area in Ale Gallons is 2.45.

	3.01 Greater Area.	7.46	
	2.00 Lesser Area.	11	{ One third of the Depth = E F.
	6.0200 (2.45 Mean Area.	746	
	4	746	
44)	202	82.06	{ The whole Content.
	176		
485)	2600		
	2425		
	175		

But the Frustum of a Cone may be gaug'd more expeditiously thus: Square the Sum of the top and bottom

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bottom Diameters; and from this Square subtract the Product of the top and bottom Diameters; multiply the Remainder, by the Depth, and divide the Product by 1077 for Ale, or 882 for Wine Gallons.

EXAMPLE.

$$26.8 = \text{Bottom Diameter.} \quad 26.8$$

$$32.9 = \text{Top Diameter.} \quad 32.9$$

$$59.7 = \text{Sum.} \quad 2412$$

$$59.7 \quad 536$$

$$804$$

$$4179$$

$$5373$$

$$2985$$

$$881.72 = \text{Product.}$$

$$3564.09$$

$$881.72$$

$$2682.37$$

$$33 = \text{Depth.}$$

$$804711$$

$$804711$$

$$1077) 88518.21 \quad (82.18. \text{ The whole Content in Ale Gall.}$$

$$8616$$

$$2358$$

$$2154$$

$$2042$$

$$1077$$

$$9651$$

$$8616$$

$$1035$$

In

In like manner may the Content of any square Pyramid be determined; the Operation being in every Respect the same, excepting that of the Divisors, which before were 1077 for Ale, and 882 for Wine Gallons, will now be 846 for Ale, and 693 for Wine Gallons.

But it has been before observed, that few Vessels, if any, are made with that Accuracy which Geometry requires; the practical and most certain Method, therefore is, to tabulate those Frustrums in the following Manner.

Let Dimensions be taken in the Middle of every 4, 6, 8 or ten Inches, as seem most convenient; and find the Areas in Gallons agreeing to those Dimensions; the Sum of all which multiplied by the Distance of the Diameters, will give the Content of the Vessel.

Now, Vessels of this Kind, being usually small, are for the most Part gaug'd by wet Inches; and therefore tabulated forward. Admit then, the Diameters be taken in the Middle of every ten Inches; for the lower ten Inches, add the Area agreeing to the Diameter, taken in the Middle of the said ten Inches, ten Times; for the second ten Inches, add the Area agreeing to its Diameter ten Times; and thus proceed to the Whole Content of the Vessel. Example (*Fig. XXI.*)

$$\text{Diameter at } \left\{ \begin{array}{l} 5 \\ 15 \\ 25 \\ 31.5 \end{array} \right\} \begin{array}{l} \text{Inches from} \\ \text{the Bottom,} \\ \text{is} \end{array} \left\{ \begin{array}{l} 27.7 \\ 29.5 \\ 31.4 \\ 32.6 \end{array} \right\} \begin{array}{l} \text{Area there-} \\ \text{of, is} \end{array} \left\{ \begin{array}{l} 2.13 \\ 2.42 \\ 2.74 \\ 2.95 \end{array} \right\} \begin{array}{l} \text{Ale} \\ \text{Gal-} \\ \text{lons.} \end{array}$$

N. B. The Diameter 31.5, is taken in the Middle of the remaining 3 Inches, at the Top.

A TABLE in ALE MEASURE.

Inches.	Ale Gallons.	Inches.	Ale Gallons.
1	2.13	18	40.66
2	4.26	19	43.08
3	6.39	20	45.50
4	8.52	21	48.24
5	10.65	22	50.98
6	12.78	23	53.72
7	14.91	24	56.46
8	17.04	25	59.20
9	19.17	26	61.94
10	21.30	27	64.68
11	23.72	28	67.42
12	26.14	29	70.16
13	28.56	30	72.90
14	30.98	31	75.85
15	33.40	32	78.80
16	35.82	33	81.75
17	38.24		

In inching Vessels, for dry Inches, it is most commodious to begin taking the Dimensions from the Top; and if there be any Fall, it must be consider'd, as is before shewn, in that Instance of a Brewer's Tun.

The Method of Examining this Table, is in this Manner.

Multiply each Area, by the Number of Inches for which it was calculated: Thus,

2.13	2.42	2.74	2.95
10	10	10	3
-----	-----	-----	-----
21.30	24.20	27.40	8.85

The Content of the lowest ten Inches, *viz.* 21.3 being compared with the Table, if found to agree, then

then add 21.3 to 24.2 the Content of the next ten Inches, and the Sum will be 45.5; which agrees with the Table at twenty Inches. Again, to 45.5 add 27.4 the Content of the next ten Inches, and the Sum will be 72.9, which agrees with the Table at thirty Inches. Again, to 72.9 add 8.85, the Content of the last three Inches; the Sum will be 81.75, the whole Content agreeable to the Table.

And here Note, that multiplying by 10 is immediately perform'd, by placing the decimal Point one more Place to the Right Hand.

But the Quantity of Liquor may expeditiously be found, without tabulating: Thus, admitting the Depth of Liquor in this Vessel to be, for Instance, 24.5 Inches.

Content of the lower ten Inches 21.3

Content of the 2d ten Inches 24.2

Content at twenty Inches 45.5.

Then multiply 2.74 the Area of each Inch, in the next ten Inches, by 4.5 the remaining Inches.

1370

1096

The Product 12.330

To this add 45.5 the Content at 20 Inches.

The Sum will be 57.83 the Quantity in the Vessel.

It is here necessary to inform the Reader, that the Distance of the Diameters, is to be measured in the Line EF (*Fig. 21.*) which being somewhat inconvenient, it will suffice to measure those Distances in the Sides AC and BD, if the Difference of the top and bottom Diameters be but small. But if the Difference be considerable, proceed thus:

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Multiply the Length of the Side, by the Distance the Diameter is taken from the Bottom, and divide the Product by the perpendicular Depth; the Quotient will be the Distance, to be measur'd on the Side.

Admit, in such a Vessel, the perpendicular Height be 50 Inches, and the Length of the Side 52 Inches; let it be required to find what Distance must be measured, on the Side, to take a Diameter at 5 Inches from the Bottom. See the Operation.

$$\begin{array}{r}
 52 = \text{the Side.} \\
 5 = \text{Distance from the Bottom.} \\
 \hline
 50 \overline{) 260} \\
 \hline
 5.2 = \text{Distance on the Side.}
 \end{array}$$

Thus it is found, that for every 5 Inches in the Perpendicular, there must be allowed 5.2 Inches on the Side.

Again, let *Fig. XXII.* be the Frustum of a Pyramid, whose Base is an Oblong, and Depth 30 Inches. By taking Dimensions in the Middle of every ten Inches, they are found to be as followeth.

$$\text{At } \left\{ \begin{array}{l} 5 \\ 15 \\ 25 \end{array} \right\} \begin{array}{l} \text{Inches from the} \\ \text{Bottom, the} \\ \text{Length is} \end{array} \left\{ \begin{array}{l} 27.5. \\ 32.5. \\ 37.5. \end{array} \right\} \begin{array}{l} \text{Breadth} \\ \text{is} \end{array} \left\{ \begin{array}{l} 22 \\ 26 \\ 30 \end{array} \right\} \begin{array}{l} \text{Area} \\ \text{is} \end{array} \left\{ \begin{array}{l} 2.14 \\ 2.99 \\ 3.98 \end{array} \right\} \text{A. G.}$$

Thus one may tabulate this Vessel, by a continued Addition of 2.14 for the lower ten Inches; of 2.99 for the next ten Inches; and of 3.98 for the upper ten Inches.

To Gauge a Sphere. (Fig. XXIII.)

Multiply the Diameter by itself, and that Product again by the Diameter; this last Product, divided

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divided by 538.5 gives Ale, or 441 gives Wine Gallons.

Diameter AB = 30

30

900

30

538.5) 27000.000 (50.13 = Content in Ale Gallons.

26925

7500

5385

21150

16155

4995

The Diameter of the Base, of the Segment of a Sphere, and Height of the said Segment being given, to find the Diameter of the Sphere. (Fig. XXIII.)

Multiply half the Diameter of the Base by itself, and divide the Product by the Segment's Height; to the Quotient, thence arising, add the Segment's Height, the Sum will be the Sphere's Diameter.

FD = 12

12

6) 144

24

6 Add.

30 = Diameter.

L

To

To Gauge the Segment of a Sphere. (Fig. XXIII.)

To once and a half, the Square of the Diameter of its Base, add twice the Square of the Segment's Height ; multiply that Sum by the Segment's Height, and then divide the Product by 1077 for Ale, or 882 for Wine Gallons.

Diameter of the Base C D = 24	Height E F = 6
24	6
—	—
96	36
48	36
—	—
2) 576	72
288	
—	
864	
72	
—	
936	
6 = Height.	

1077) 5616.00 (5.21 = Content in Ale Gall.
5385
—
2310
2154
—
1560
1077
—
483

But if the Diameter of the Sphere, and Height of the Segment be given: From six times the Diameter of the Sphere, subtract four times the Height of the Segment ; then multiply the Remainder by the

the Square of the Height, and divide the Product by 1077 for Ale, or 882 for Wine Gallons.

Diameter AB = 30	Height EF = 6
6	4
---	---
180	24
24	

156	
36 = Square of the Height.	

$$\begin{array}{r} 936 \\ 468 \\ \hline \end{array}$$

1077) 5616 (5.21 = Content as above.

$$\begin{array}{r} 5385 \\ \hline 2310 \\ 2154 \\ \hline 1560 \\ 1077 \\ \hline 483 \end{array}$$

To Gauge COPPERS.

To Gauge a Copper, (Fig. XXIV.) First cover the Crown with Liquor, and then take Diameters, at the Middle of every 4, 6, 8, or 10 Inches, as seem most requisite; and tabulate as, is before taught, in the Frustum of a Cone.

But inasmuch as Coppers, when fix'd, are usually so set, that the Liquor may conveniently run off; the Cock in order thereto, is set somewhat the lowest, whereby the Liquor's Surface becomes Elliptical: The Method therefore of Gauging a Copper, when fix'd, is as follows.

First, cover the Crown with Liquor; and admit 93 Gallons to cover the Crown; the Cock being at

L 2

D,

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D, the Surface of the Liquor touching at *a* and *o*. With a String, take the Distance from *o* to B, and set the same Distance from *a* to A, opposite to the Cock; the same Distance also set off, from the Liquor's Surface, on each Side, at half the Distance between A and B; then extending a Line from B to A, take with a Rule the nearest Distance from D, to the Line BA, *viz.* DE, supposing it 44 Inches; from this Depth, subtract the Depth of the Liquor, that covers the Crown, supposing it 8 Inches; the Remainder 36 Inches is the neat Depth: Now admitting the Diameters, to be taken in the Middle of every 6 Inches, make Marks on the Rule DE, at 3, 9, 15, 21, 27, and 33 Inches from the Surface of the Liquor; and set off those Distances upon the Side of the Copper, at the Marks *b, d, f, h, k, m*, parallel to the Liquor's Surface; do the same upon the opposite Side at *c, e, g, i, l, n*; the same Distances also set off, at the Middle between those Places on each Side; then take the Diameters *bc, de, fg, &c.* as also their cross Diameters, at right Angles as follows.

				Ale Gall.	B. F. G.
<i>bc</i> = 93.5	Cross Diameters are.	95.3	Mean Diameters are.	24.29	or 0 2 6.29
<i>de</i> = 97.6		97.2		26.42	
<i>fg</i> = 101.6		101.2		28.63	
<i>hi</i> = 105.7		103.7		30.94	
<i>kl</i> = 109.6		108.8		33.21	
<i>mn</i> = 113.4		112.8		35.62	
				Sum	4 3 8.11
				Multiplied by	6, the
Distance of the Diameters.					
				To cover the Crown =	29 3 3.66 } Add.
					2 2 3
				Whole Content =	32 1 6.66

Note, The Difference of the transverse, and conjugate Diameters being but small; the Mean Diameter is nearly half the Sum of the two Diameters, as is shewn above.

To

To Tabulate the Copper for the dry Inches.

From the whole Content, subtract the Area of each Inch of the upper six Inches six times; then from the Remainder, subtract the Area of each Inch, of the next six Inches six Times; and thus proceed to the Crown, as follows.

Dry Inch. Full.	B.	F.	G.	Inch.	B.	F.	G.	Inch.	B.	F.	G.
1	32	1	6.66	13	20	0	3.74	25	10	1	1.84
2	31	1	7.04	14	19	0	8.80	26	9	2	2.42
3	30	1	7.42	15	18	1	4.86	27	8	3	3.00
4	29	1	7.80	16	17	2	0.92	28	8	0	3.58
5	28	1	8.18	17	16	2	5.98	29	7	1	4.86
6	27	1	8.56	18	15	3	2.04	30	6	2	4.74
Subtracting o 3 8.62.				Subtracting o 3 3.94.				Subtracting o 2 8.42.			
7	25	2	2.73	19	15	0	0.41	31	5	3	7.45
8	24	2	5.52	20	14	0	7.78	32	5	1	1.16
9	23	2	8.31	21	13	1	6.15	33	4	2	3.87
10	22	3	2.10	22	12	2	4.52	34	3	3	6.58
11	21	3	4.89	23	11	3	2.89	35	3	1	0.29
12	20	3	7.68	24	11	0	1.26	36	2	2	3.00
Subtracting o 3 6.21.				Subtracting o 3 1.63.				Subtracting o 2 6.29.			

To Tabulate this Copper, for the wet Inches.

In tabulating a Copper for the wet Inches, it is necessary in covering the Bottom, to pour in Liquor, at your dipping Place to even Inches, before the Diameters be taken; which Depth is here 8 Inches, and the Liquor at that Depth 93 Gallons, or 2 B. 2 F. 3 G. to which are to be added, the Areas beginning at the Bottom, after the same Method as, is before taught, in the Frustrum of a Cone.

Inch.	B.	F.	G.
Crown 8	2	2-3	
9	3	1 0.29	
10	3	3 6.58	
11	4	2 3.87	
12	5	1 1.16	
13	5	3 7.45	
14	6	2 4.74	
		Adding o 2 6.29.	
15	7	1 4.16	
16	8	0 3.58	
17	8	3 3.00	
18	9	2 2.42	
19	10	1 1.84	
20	11	0 1.26	
		Adding o 2 8.42.	
Inch.	B.	F.	G.
	21	11 3 2.89	
	22	12 2 4.52	
	23	13 1 6.15	
	24	14 0 7.78	
	25	15 0 0.41	
	26	15 3 2.04	
		Adding o 3 1.63.	
	27	16 2 5.98	
	28	17 2 0.92	
	29	18 1 4.86	
	30	19 0 8.80	
	31	20 0 3.74	
	32	20 3 7.68	
		Adding o 3 3.94.	
Inch.	B.	F.	G.
	33	21 3 4.89	
	34	22 3 2.10	
	35	23 2 8.31	
	36	24 2 5.52	
	37	25 2 2.73	
	38	26 1 8.94	
		Adding o 3 6.21.	
	39	27 1 8.56	
	40	28 1 8.18	
	41	29 1 7.80	
	42	30 1 7.42	
	43	31 1 7.04	
	44	32 1 6.66	
		Adding o 3 8.62.	

The Quantity of Liquor, which will cover the Crown, may be nearly found thus: Add the Diameter, at the Top of the Crown, to the Diameter, at the Bottom of the Crown, and multiply the Sum, by the top Diameter, and reserve the Product; then add half the Square of the bottom Diameter, to twice the Square of the Height, and subtract this Sum from the Product, before reserved; multiply the Remainder by the Height, and divide by 1077 for Ale, or 882 for Wine Gallons.

After the same Manner, Stills may be gaug'd, and tabulated.

OF CASK GAUGING.

Cask Gauging, of all that happens in Practice, is the most difficult and uncertain, by Reason of the Uncertainty of the Curvature of the Cask: This Curvature, an expert Gauger may nearly judge of by the Eye only; but I shall hereafter lay down such Rules, as will infallibly direct the Learner to this Knowledge; and first of all shew the practical, and most correct Methods of taking Dimensions. When Casks are full, you can seldom attain all their Dimensions. In a standing Cask, the Bung Diameter, is most difficult to be attained; in a lying Cask, the Length. Wherefore,

To take Dimensions in a lying CASK.

Take the Head Diameter, close to its Outside, and add for small Casks 3 Tenths; for Casks of thirty, forty, or fifty Gallons 4 Tenths; for larger Casks 5 or 6 Tenths of an Inch; the Sum is very near the Head Diameter within.

In taking the Bung Diameter observe, by moving the Rule to and fro, whether the Staff opposite to the Bung, be thicker, or thinner than the rest; and if it be, make an Allowance accordingly. The Length is
most

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most commodiously taken with Callipers made for that Purpose, allowing for the Thickness of both Heads 1.0, or 1.5, or 2.0 Inches, according to the Size of the Cask. But if you have no Callipers, do thus: Measure the Length of the Staff; then laying a Rule across each Head, measure the Depth of the Chimes; which together with the Thickness of the Heads, being subtracted from the Length of the Staff, leaves the Length within.

To take Dimensions of a standing CASK.

The Head is to be taken as before; the Bung Diameter, is best taken thus: Lay one Side of a Square over the Chimes of the Cask, that the other may just touch the Bung of the Cask, (A Square of this Sort may be had of Mr. Gilbert, Mathematical Instrument-Maker on Tower-Hill,) then observe what each Chime of the Cask cuts on the Square; add the Inches and Tenths, which the Chime opposite the Square cuts, to the Inches and Tenths, which the nearer Chime cuts; and from the Sum subtract twice the Thickness of the Bung Staff, viz. 0.8, or 1.0, or 1.2, or 1.4, or 1.6, according to the Size of the Cask.

The Length is to be taken with the same Caution, that the Bung Diameter is in a lying Cask.

The Dimensions thus taken, the next Consideration is the Variety of the Curvature of the Cask. The outside Lines, (Fig. XXV.) represent a Cask, which is the middle Frustum of a Spheroid; and this Cask hath the greatest Curvature of any regularly made. The prick'd Lines represent a Cask, composed of the Frustums of two Cones abutting upon one common Base; and of these none I am certain were ever made: However, all the Varieties of Casks lie between these two; and of these Varieties, the Spheroid being esteem'd none, we reckon but three; and the Method of finding how the Cask is to be gaug'd, is thus:

Making

Making a Mark exactly at the Middle of the Staff, and laying the Square over the Chimes to touch the Mark, observe the Distance from the Chimes, to this Mark at the Bung; and at half that Distance from the Chime, measure with a Rule the Distance of the Side of the Square from the Staff, holding the Rule parallel to the upper Side of the Square; then divide that Distance, by the Distance the Chime cuts on the Side of the Square, and if the Quotient be about .23 the Cask is Spheroidical; if it be about .28 the Cask is of the first Variety; if about .34 of the second Variety; if about .42 of the third Variety. And by this Means, if that Nicety be required, even the Varieties themselves may be truly divided.

Now admit (Fig. XXV.) the Length to be 46 Inches; the Head Diameter 25 Inches, and the Bung Diameter 31 Inches.

Suppose that laying the Square over the Chimes, the Square touches the Chime at 3.5 Inches, and exactly at half Way, between the Chime and the Mark at the Bung, the Distance of the Side of the Square, from the Staff is .8.

$$\begin{array}{r}
 3.5 \overline{) .800} \quad (.22 \\
 \underline{70} \\
 100 \\
 \underline{70} \\
 30
 \end{array}$$

Here the Quotient being near .23, it may be concluded, that the Cask is the Frustum of a Spheroid; which may be gauged thus.

To twice the Square of the Bung, add once the Square of the Head; multiply this Sum by the Length, and divide by 1077 for Ale, or 882 for Wine Gallons.

M

Head

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Head = 25

25

—

125

50

—

625

Bung = 31

31

—

31

93

—

961 = Square of the Bung.

961

625 = Square of the Head.

2547

46 = Length.

15282

10188

1077) 117162 (108.7 Ale Gallons.

1077

9462

8616

—

8460

7539

—

921

Now supposing a Cask of the same Dimensions,
and at the Place where the Distance was found to
be .8, is now found to be one Inch.

3.5) 1.000 (.28

70

—

300

280

—

20

Here

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Here the Quotient being .28, one may conclude the Cask to be of the first Variety; which is to be gauged thus.

To once and three Quarters of the Square of the Bung, add once the Square of the Head, and one fourth Part of the Product of the two Diameters; this Sum multiply by the Length, and divide the Product by 1077 for Ale, or 882 for Wine Gallons.

$$\begin{array}{r}
 31 = \text{Bung.} \\
 25 = \text{Head.} \\
 \hline
 155 \\
 62 \\
 \hline
 4) 775 \\
 \hline
 193.75
 \end{array}$$

$$\begin{array}{l}
 \text{Square of the Bung} = 961 \\
 \text{Half thereof} = 480.5 \\
 \text{A Fourth thereof} = 240.25 \\
 \text{Square of the Head} = 625 \\
 \text{A Fourth of the Product} = 193.75 \\
 \text{Sum} = 2500.50 \\
 \quad \quad 46 = \text{Length.}
 \end{array}$$

$$\begin{array}{r}
 1500300 \\
 1000200 \\
 \hline
 1077) 115023.00 \text{ (106.79 Ale Gall.} \\
 \underline{1077} \\
 7323 \\
 \underline{6462} \\
 8610 \\
 \underline{7539} \\
 10710 \\
 \underline{9693} \\
 1017
 \end{array}$$

M 2

Again,

Again, supposing a Cask of the same Dimensions, and the Distance of the Square, from the Staff, at the aforesaid Place, to be 1.2.

$$\begin{array}{r}
 3.5) 1.200 (.34 \\
 \underline{105} \\
 150 \\
 \underline{140} \\
 10
 \end{array}$$

Here the Quotient being .34, one may judge the Cask to be of the second Variety; which may be gauged thus.

To once and half the Square of the Bung, add once the Square of the Head, and half the Product of the Diameters; this Sum multiply by the Length, and divide the Product by 1077 for Ale, or 882 for Wine Gallons.

$$\begin{array}{r}
 31 = \text{Bung.} \\
 25 = \text{Head.} \\
 \hline
 155 \\
 62 \\
 \hline
 2) 775 \\
 \hline
 387.5
 \end{array}$$

Square

$$\begin{aligned}\text{Square of the Bung} &= 961 \\ \text{Half thereof} &= 480.5 \\ \text{Square of the Head} &= 625 \\ \text{Half the Product} &= 387.5\end{aligned}$$

$$\begin{aligned}\text{Sum} &= 2454.0 \\ \text{Length} &= 46\end{aligned}$$

$$\begin{array}{r} 14724 \\ 9816 \\ 1077) 112884.00 \text{ (104.8 Ale Gall.} \\ 1077 \end{array}$$

$$\begin{array}{r} 5184 \\ 4308 \end{array}$$

$$\begin{array}{r} 8760 \\ 8616 \end{array}$$

$$144$$

Again, supposing a Cask of the same Dimensions, and the Distance of the Square, at the afore-said Place, to be 1.5.

$$\begin{array}{r} 3.5) 1.500 \text{ (.42.} \\ 140 \\ \hline 100 \\ 70 \\ \hline 30 \end{array}$$

Here

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Here the Quotient being .42, it may be concluded to be of the third Variety; which is to be gauged thus.

To once and a Fourth of the Square of the Bung, add the Square of the Head, and three Fourths of the Product of the Diameters; multiply this Sum by the Length, and divide the Product by 1077 for Ale, or 882 for Wine Gallons.

$$\begin{array}{r}
 31 = \text{Bung.} \\
 25 = \text{Head.} \\
 \hline
 155 \\
 62 \\
 \hline
 775 \\
 3 \\
 \hline
 4) 2325 \\
 \hline
 581.25
 \end{array}$$

Square of the Bung = 961

A Fourth thereof = 240.25

Square of the Head = 625

Three 4ths of the Product = 581.25

Sum = 2407.50
Length = 46

144450
96300

1077) 110745.0 (102.8 Ale Gall.
1077

3045
2154

8910
8616

294

Now,

Now, these Methods being (as I believe) new, I shall, to avoid Cavilling, lay down their Investigation. Suppose, then, the Difference between the Content of the Fruustum of a Spheroid, and of two Cones abutting upon one common Base, the Diameters and Lengths being alike, to be divided into four equal Parts ; then let

D = Bung Diameter.

d = Head Diameter.

Then a fourth Part of the Difference of the triple Areas of those Casks, (the common Factor .7854 being rejected out of each Term) will be $\frac{DD - Dd}{4}$; which being subtracted three times

successively, from the triple Area of the spheroidal Cask, viz. from $2 DD + dd$ (where also the common Factor .7854 is rejected out of each Term) will leave at the first Subtraction,

1.75 DD + .25 Dd + dd, for the first Variety.

Secondly, 1.5 DD + .5 Dd + dd, for the second Variety.

Thirdly, 1.25 DD + .75 Dd + dd, for the third Variety.

Lastly, DD + Dd + dd, for the Frustrums of two Cones.

Or the Contents of Casks may be found thus : Take the Difference of the Head and Bung Diameters ; and seek for this Difference, in the following Table, according to the Variety of the Cask, looking for the Inches on the left-hand Side of the Table, and the Tenths at the Top ; and in the Angle of Meeting is a Number, which is to be added to the Head Diameter, the Sum is the Mean Diameter ; which being multiplied by itself, and the Product thence arising, by the Length, this last Product being divided by 359 will give Ale, or by 294 will give Wine Gallons.

Let

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Let there be a Cask of the second Variety, whose Head Diameter is 24.8, the Bung Diameter 31.2, and the Length 43 Inches.

$$\text{Bung} = 31.2$$

$$\text{Head} = 24.8$$

$$\underline{6.4} = \text{Difference of Diameters.}$$

Now in the Table of the second Variety, against 6 Inches, and under 4 Tenths, will be found 3 84, which added to 24.8, the Head Diameter, the Sum is 28.64, the Mean Diameter; then the Work is as follows.

$$\begin{array}{r} 28.64 \\ 28.64 \\ \hline 11456 \\ 17184 \\ 22912 \\ 5728 \\ \hline 820.2496 \end{array}$$



$$\underline{43} = \text{Length.}$$

$$\begin{array}{r} 24607488 \\ 32809984 \\ \hline 294) 35270.7328 \end{array} \quad (119.96 \text{ Wine Gall.}$$

$$\underline{294}$$

$$587$$

$$\underline{294}$$

$$2930$$

$$\underline{2646}$$

$$2847$$

$$\underline{2646}$$

$$2013$$

$$\underline{1764}$$

$$249$$

SPHE-

S P H E R O I D.

	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9
1	0.67	0.74	0.81	0.87	0.94	1.01	1.08	1.15	1.21	1.28
2	1.35	1.42	1.49	1.56	1.62	1.69	1.76	1.83	1.90	1.97
3	2.04	2.11	2.17	2.24	2.31	2.38	2.45	2.52	2.59	2.66
4	2.73	2.80	2.87	2.94	3.01	3.08	3.15	3.22	3.29	3.36
5	3.43	3.50	3.57	3.64	3.71	3.79	3.86	3.93	4.00	4.07
6	4.14	4.21	4.28	4.35	4.42	4.49	4.56	4.63	4.71	4.78
7	4.85	4.92	4.99	5.06	5.14	5.21	5.28	5.35	5.42	5.49
8	5.57	5.64	5.71	5.78	5.85	5.93	6.00	6.07	6.14	6.21
9	6.29	6.36	6.43	6.50	6.57	6.65	6.72	6.79	6.86	6.93
10	7.00	7.07	7.14	7.21	7.29	7.36	7.44	7.51	7.58	7.65

I. V A R I E T Y.

1	0.63	0.69	0.75	0.82	0.88	0.95	1.01	1.07	1.14	1.20
2	1.27	1.33	1.39	1.46	1.52	1.59	1.65	1.71	1.78	1.84
3	1.91	1.97	2.04	2.10	2.17	2.23	2.30	2.36	2.43	2.49
4	2.56	2.62	2.69	2.75	2.82	2.88	2.95	3.01	3.08	3.14
5	3.21	3.27	3.34	3.40	3.47	3.54	3.60	3.67	3.73	3.80
6	3.87	3.93	4.00	4.06	4.13	4.20	4.26	4.33	4.39	4.46
7	4.53	4.59	4.66	4.72	4.79	4.86	4.92	4.99	5.05	5.12
8	5.19	5.25	5.32	5.38	5.45	5.52	5.58	5.65	5.71	5.78
9	5.85	5.91	5.98	6.04	6.11	6.18	6.24	6.31	6.37	6.44
10	6.51	6.57	6.64	6.70	6.77	6.84	6.90	6.96	7.03	7.10

II. V A R I E T Y.

1	0.59	0.64	0.70	0.76	0.82	0.88	0.94	1.00	1.06	1.12
2	1.18	1.24	1.30	1.36	1.42	1.48	1.54	1.60	1.66	1.72
3	1.78	1.84	1.90	1.96	2.02	2.08	2.14	2.20	2.26	2.32
4	2.38	2.44	2.50	2.56	2.62	2.68	2.74	2.80	2.86	2.92
5	2.99	3.05	3.11	3.17	3.23	3.29	3.35	3.41	3.47	3.53
6	3.60	3.66	3.72	3.78	3.84	3.90	3.96	4.02	4.08	4.14
7	4.21	4.27	4.33	4.39	4.45	4.51	4.57	4.63	4.69	4.75
8	4.82	4.88	4.94	5.00	5.06	5.12	5.18	5.24	5.30	5.36
9	5.43	5.49	5.55	5.61	5.67	5.73	5.80	5.86	5.92	5.99
10	6.05	6.11	6.17	6.23	6.30	6.36	6.42	6.48	6.54	6.60

III. V A R I E T Y.

1	0.55	0.60	0.66	0.71	0.77	0.82	0.88	0.93	0.99	1.04
2	1.10	1.15	1.21	1.26	1.32	1.37	1.43	1.48	1.54	1.59
3	1.65	1.70	1.76	1.81	1.87	1.92	1.98	2.03	2.09	2.14
4	2.20	2.25	2.31	2.36	2.42	2.48	2.53	2.59	2.64	2.70
5	2.76	2.81	2.87	2.92	2.98	3.04	3.09	3.15	3.20	3.26
6	3.32	3.37	3.43	3.48	3.54	3.60	3.65	3.71	3.76	3.82
7	3.88	3.93	3.99	4.04	4.10	4.16	4.21	4.27	4.32	4.38
8	4.44	4.50	4.55	4.61	4.66	4.72	4.78	4.84	4.89	4.95
9	5.01	5.07	5.12	5.18	5.24	5.30	5.36	5.41	5.47	5.53
10	5.59	5.65	5.70	5.76	5.82	5.88	5.94	5.99	6.05	6.11

Or the Number to be added to the Head Diameter, may be nearly found, without the Table, thus: Multiply the Difference between the Head and Bung Diameter by .7, for a Spheroid; by .65 for the first Variety; by .6 for the second Variety; or by .55 for the third Variety. In the last Example, where the Cask was supposed of the second Variety, the Difference of the Diameters was 6.4; this multiplied by .6, the Product is 3.84, the same Number, as was found in the Table.

To make the Learner more compleat in this curious Part of Gauging, take another Example.

Let there be a Cask, whose Length is 30.5, the Head Diameter 22.5, and the Bung Diameter 26.8: Now suppose, as is before taught, the Chime cuts the Side of the Square at 2.5, and at the Middle between the Chime, and the Bung, the Distance be .8

$$\begin{array}{r}
 2.5) .800 \text{ (.32} \\
 \underline{75} \\
 50 \\
 \underline{50} \\
 ..
 \end{array}$$

Now .32 being the Quotient, one may perceive the Cask to be neither of the first nor of the second Variety, but between both: Therefore, let not the Difference of the Diameters be multiplied by .65, nor yet by .6; but by .63: Then the Operation will be as follows.

$$\begin{array}{r}
 26.8 \\
 22.5 \\
 \hline
 4.3 \\
 .63 \\
 \hline
 129 \\
 258 \\
 \hline
 2.709 \\
 \text{Add } 22.5 \\
 \hline
 \text{Mean Diameter} = 25.209
 \end{array}$$

$$\begin{array}{r}
 25.2 \\
 25.2 \\
 \hline
 504 \\
 1260 \\
 504 \\
 \hline
 635.04 \\
 30.5 = \text{Length.} \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 317520 \\
 1905120 \\
 \hline
 359) 19368.720 \text{ (53.95 Ale Gallons.)} \\
 1795 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 1418 \\
 1077 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 3417 \\
 3231 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 1862 \\
 1795 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 67 \\
 \text{N } 2
 \end{array}$$

And

And thus a Cask may be gauged, to an incredible Nicety: which Observations, altho' no Author that I know of, ever yet published, I was not willing that my quondam Brethren, or the young Tyro, hereafter, should be unacquainted therewith.

To Ullage a Cask; that is, to find the Quantity of Liquor in a Cask partly full.

This admits of two Cases, viz. one to find the Quantity of Liquor in a Cask lying with its Axis parallel to the Horizon; the other to find the Quantity of Liquor in a Cask standing with its Axis perpendicular to the Horizon.

Let it, in the first Place, be required to Ullage a lying Cask.

By the Table, or otherwise, find the mean Diameter; then let half the Difference between it, and the Bung Diameter be subtracted from the wet Inches; the Remainder, for Plainness Sake, let be called the net Wet; then multiply this net Wet by .6, and subtract the Product from the mean Diameter; this Remainder multiply by the net Wet, and extract the square Root of the Product; this Root multiply by the net Wet, the Product thence arising, by the Length of the Cask, and divide by 212 for Ale, or 173 for Wine Gallons.

If a Cask be the middle Fruustum of a Spheroid, whose Head Diameter is 25, Bung Diameter 31, Length 46 Inches, and the wet Inches 9; it is required to find the Quantity of Liquor remaining in the Cask.

From 31 the Bung, subtract the mean Diameter 29.14, the Remainder will be 1.86; half thereof being subtracted from 9, the wet Inches, leaves 8.07 for the net Wet; which multiplied by .6, gives 4.842; this taken from 29.14, the mean Diameter, leaves 24.291; which multiplied by 8.07, the

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the net Wet, gives 196.08486; the Square Root of which being extracted, gives 14.003 : Wherefore,

$$14.003 = \text{Root.}$$

8.07

98021

1120240

113.00421

46 = Length.

6780.2526

45201684

212) 5198.19366 (24.519 Ale Gallons.

424

958

848

1101

1060

419

212

2073

1908

165

If the Cask happens to be above half full, proceed in the very same Manner, with the dry Inches, as was done before with the wet; and the Result, will be the Quantity of Liquor the Cask wants of being full; which subtracted from the whole

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whole Content, leaves the Quantity of Liquor in the Cask.

To Ullage a standing CASK.

Find the Diameter at the Surface of the Liquor, thus: First find the Middle of the Cask by girting the Staff with a String, and double that String, by which means the Middle of the Cask, at the Outside of the Staff, may be found; there make a Mark, and from this Mark set off the Distance of the Liquor's Surface from the Bung upwards, and there make another Mark: Then laying the Side of a Square over the Chime of the Cask, that the other Side may just touch the Bung, measure the Distance between the Side of the Square and this last Mark, holding your Rule parallel to the Head of the Cask; the Double of that Distance, subtracted from the Bung Diameter, leaves the Diameter of the Liquor's Surface: Now between these two Diameters, find a mean Diameter: The Area of which multiplied by the Liquor's Distance from the Bung, shews how much the Cask is above half full, if the Liquor be above the Bung; or how much less than that half, if it be below the Bung. Now, suppose the former Cask standing upright, and the Depth of the Liquor to be 9, or 37 Inches; both which are 14 Inches distant from the Bung: (*Fig. XXV.*) Having found the Middle of the Cask at A, set off 14 Inches to B; then laying the Square over the Chime, that the other side may just touch A, measure the Distance BC, and it will be found to be 1.05; which doubled is 2.1; this subtracted from 31 the Bung Diameter, leaves 28.9, the Diameter at the Liquor's Surface: Now, seeking 2.1, the Difference of the Diameters, in its proper Table,
against

againſt that will be found 1.42; which added to 28.9, the Sum is 30.32, the mean Diameter.

$$\begin{array}{r} 30.32 \\ 30.32 \\ \hline 6064 \\ 9096 \\ 90960 \end{array}$$

359) 919.3024 (2.5607 = Area in Beer Gallons.
718 14 = Distance from the Bung.

$$\begin{array}{r} 2013 \\ 1795 \\ \hline \end{array} \quad \begin{array}{r} 102428 \\ 25607 \\ \hline \end{array}$$

$$\begin{array}{r} 2180 \\ 2154 \\ \hline \end{array} \quad \begin{array}{r} 35.8498 \\ 54.35 \\ \hline \end{array} \quad \text{Add } \frac{1}{2} \text{ the Content.}$$

$$\begin{array}{r} 2624 \\ 2513 \\ \hline \end{array} \quad 90.1998 = \text{Content at 37 Inches.}$$

III

$$\begin{array}{r} 54.35 = \text{Half Content.} \\ 35.84 \text{ Subtract.} \\ \hline \end{array}$$

18.51 Content at 9 Inches.

These laſt Methods, though very exact, are ſome-what operoſe; it may not therefore be amiſs to lay down two other Methods for the ſame Purpoſe; which nearly agree to the Lines of Segments, on the Sliding-Rule.

For a lying C A S K.

If the Caſk be not half full, work with the wet Inches; but if above half full, work with the dry

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dry Inches, to find how much it wants of being full: Now, divide the wet or dry Inches, as the Case requires, by the Bung Diameter, to three Places of Decimals, and seek the Quotient in the following short Table; which may be soon learn'd by Heart; and opposite to the Quotient will be found a Number, by which multiply the Quotient, and the Product is the Segment; this Segment multiplied by the whole Content, shews the Quantity of Liquor, or Wants of the Cask.

A T A B L E.

Quotient.	Multiplier.
.04	— .2
.07	— .3
.09	— .4
.14	— .5
.20	— .6
.24	— .7
.31	— .8
.40	— .9
.50	— 1.0

Let the Cask be the same as before, whose wet Inches were 9.

Bung.	Wet.	Quotient.
31)	9.000	(.290
	62	
	—	
	280	
	279	
	—	
	10	

Here seek in the Table for the Quotient .29, and the nearest below it will be found .24, and the nearest above it .31; against which Numbers are

7 and 8 ; then consider that as 7, the Difference between 24 and 31 gives 10, so 5 the Difference between 24 and 29 will give about 7 ; therefore take .77 for the Multiplier.

$$\begin{array}{r}
 .290 \\
 .77 \text{ Multiplier.} \\
 \hline
 2030 \\
 2030 \\
 \hline
 .22330 \\
 108.7 = \text{Whole Content.} \\
 \hline
 156310 \\
 178640 \\
 223300 \\
 \hline
 24.272710 = \text{Liquor in the Cask.}
 \end{array}$$

For a standing C A S K.

Divide the wet Inches by the Length, to three Places of Decimals in the Quotient ; if the Quotient be under .250, multiply it by .9 ; if it be between .250 and .750, multiply it by 1.1, and from the Product subtract .05 ; if it be above .750 multiply it by .9, and to the Product add .100 : This Number multiplied by the whole Content of the Cask, shews the Quantity of Liquor in the Cask.

O

Length

Length.	Wet.	
46)	37.000	(.804
	368	.9
	-----	-----
	200	.7236
	184	.100
	-----	-----
	16	.8236
		108.7 = Whole Content.

		57652
		65888
		82360

		89.52532 = Quantity in the Cask.

These last Methods are very concise, and for their Ease and Exactness are very valuable.

Some useful PROBLEMS.

PROBLEM I.

The Content and Depth, of a Cylinder being given; to find the Diameter.

Multiply the Content by 359 if Ale Gallons, or 294 if Wine Gallons, and divide the Product by the Depth; the square Root of the Quotient is the Diameter.

PROBLEM II.

The Content, and Diameter, of a Cylinder being given; to find the Depth.

Multiply the Content by 359 if Ale Gallons, or 294 if Wine Gallons, and divide the Product by the Square of the Diameter; the Quotient is the Depth.

PRO-

PROBLEM III.

The Content of a Cylinder, and the Proportion the Diameter hath to the Depth, being given; to find the Diameter and Depth.

Multiply the Content by 359 for Ale Gallons, or 294 for Wine Gallons, and this Product multiply by the Proportion of the Diameter, and this last Product divide by the Proportion of the Depth; the Cube Root of the Quotient will be the Diameter; then multiply the Diameter by the Proportion of the Depth, and divide the Product by the Proportion of the Diameter, the Quotient will be the Cylinder's Depth.

PROBLEM IV.

The Content, Depth, and Difference of the Diameters, of a conical Tun being given; to find the Diameters.

Multiply the Content by 359 if Ale Gallons, or 294 if Wine Gallons; divide the Product by the Depth, and reserve the Quotient; then multiply the Difference of the Diameters by itself, and divide the Product by 12; the Quotient subtract from that before reserv'd, and extract the square Root of the Remainder; to this Root add half the Difference of the Diameters, the Sum is the greater Diameter; or from the Root subtract half the Difference, the Remainder is the lesser Diameter.

PROBLEM V.

The Content, Depth, and Proportion of the Diameters of a conical Tun being given; to find the Diameters.

Divide the Proportion of the greater Diameter, by the Proportion of the lesser Diameter, and to
O 2 the

the Quotient, add the Square of itself, and one more; multiply this Sum by the Depth, and reserve the Product for a Divisor; then multiply the Content by 1077 if Ale, or 882 if Wine Gallons, and divide the Product by the aforesaid Divisor; the square Root of the Quotient is the lesser Diameter; which multiplied by the greater Proportion, and the Product divided by the lesser Proportion, gives the greater Diameter.

PROBLEM VI.

The Content, Proportion of the Diameters, and Depth of a conical Tun being given; to find the Diameters and Depth.

Divide the greater Proportion of the Diameters, by the lesser, and to the Quotient add the Square of itself, and one more; this Sum multiply by the Proportion of the Depth, and divide the Product by the Proportion of the lesser Diameter, and reserve the Quotient for a Divisor; then multiply the Content by 1077 if Ale, or 882 if Wine Gallons, and divide the Product by the Divisor before reserv'd: The Cube Root of the Quotient will be the lesser Diameter. The other Dimensions are found, by multiplying the lesser Diameter by their Proportions, and dividing the Product by the Proportion of the lesser Diameter.

PROBLEM VII.

The Area of a Superficies being given, to find its Dimensions, in the same Proportion, as those of another given Superficies.

Multiply the Area, whose Dimensions are required, by the Square of any one of the given Dimensions, and divide the Product by the Content

tent of that, whose Dimensions are given; the square Root of the Quotient will be the Dimension corresponding to that, by whose Square you multiplied.

PROBLEM VIII.

The Dimensions, and Content of a Solid being given; to find the Dimensions of another Solid, whose Content is also given, in the same Proportion.

Multiply the Content, whose Dimensions are required, by the Cube of any one of the given Dimensions, and divide the Product by the Content whose Dimensions are given; the Cube Root of this Quotient is the Dimension, corresponding to that, by whose Cube you multiplied.

PROBLEM IX.

The Content, Length, Difference of the Diameters, and Variety of a Cask being given; to find the Diameters.

Multiply the Content by 359 if Ale, or 294 if Wine Gallons, and divide the Product by the Length, and extract the square Root of the Quotient; from this Root subtract the Number corresponding to the Difference of the Diameters, taken out of its proper Table, the Remainder will be the Head Diameter.

PROBLEM X.

The Content, Length, Proportion of the Diameters, and Variety of a Cask being given; to find the Diameters.

Multiply the Content of the Cask by 359 if Ale, or 294 if Wine Gallons, and divide the Product by the Length; the square Root of the Quotient

tient will be the mean Diameter of the Cask. Now find a mean Diameter between the given Proportion; then multiply the mean Diameter of the Cask, by the Mean of the given Proportions, and divide by the Proportion of the Diameter sought; the Quotient is the Diameter sought.

PROBLEM XI.

The Content, Proportion of the Diameters, and of the Length, and the Variety being given; to find the Diameters and Length.

Find a mean Diameter, between the Proportions of the Diameters; multiply this Mean by itself, and the Product multiply by the Proportion of the Length; this last Product reserve for a Divisor. Now multiply the Content by 359 if Ale, or 294 if Wine Gallons, and divide the Product by the Divisor before reserv'd; then extract the Cube Root of the Quotient, this Root multiplied by the given Proportions separately, will produce the corresponding Dimensions.

PROBLEM XII.

The Content, Head Diameter, Length and Variety being given; to find the Bung Diameter.

Multiply the Content by 359 for Ale, or 294 for Wine Gallons, and divide the Product by the Length; then extract the square Root of the Quotient; from this Root, subtract the Head Diameter; the Remainder divided by .7 for a Spheroid, or by .65 for the first Variety, or by .6 for the second Variety, or by .55 for the third Variety, gives the Difference of the Diameters; which added to the Head Diameter, gives the Bung Diameter.

PROBLEM XIII.

The Content, Bung Diameter, Length and Variety being given; to find the Head Diameter.

Proceed as in the last Problem, so far as the square Root, and subtract this Root from the Bung Diameter; the Remainder divided by .3 for a Spheroid, or by .35 for the first Variety, or by .4 for the second Variety, or by .45 for the third Variety, gives the Difference of the Diameters; which subtracted from the Bung Diameter, leaves the Head Diameter.

PROBLEM XIV.

The Content, Head and Bung Diameters, and Variety being given; to find the Length.

First find a mean Diameter, and multiply it by itself for a Divisor; then multiply the Content by 359 for Ale, or 294 for Wine Gallons; and this Product divide by the former Divisor; the Quotient will be the Length.

The Construction of the Diagonal Line.

The Contents of homogeneous Solids, are in the same Proportion, as the Cubes of their Dimensions; that is, as the Content of any Solid, is to the Cube of any one of its Dimensions; so is the Content of any other like Solid, to the Cube of the corresponding Dimension.

Now this Line is constructed for a Cask, whose Content is 60 Ale Gallons, and the Diagonal thereof 30 Inches; therefore the Proportion holds: As 60, is to the Cube of 30; so is any other Content, to the Cube of the Diagonal; that is, multiply the Content by 450, the Cube Root of the Product is the Diagonal.

Let

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Let it be required to find the Diagonal for 20 Ale Gallons.

$$\begin{array}{r}
 450 \\
 20 \\
 \hline
 60 \overline{) 9000} \quad (\begin{array}{l} 20 \text{ Supposed Root.} \\ 3 \end{array} \quad \begin{array}{l} 20 \\ 20 \\ \hline 400 \end{array} \\
 \hline
 150 \quad 60 \text{ Divisor.} \quad 12 \overline{) 400} \\
 \hline
 33.33 \quad 10.8 \\
 \hline
 116.67 \quad 10 \text{ Half supposed Root.} \\
 \hline
 1 \quad 208 \overline{) 01667} \quad 20.8 \text{ True Root and Diagonal sought.} \\
 \hline
 1664 \\
 \hline
 3
 \end{array}$$

Thus we find that 20 Gallons on the Diagonal Line, must be set at 20 Inches and 8 Tenths.

But hence it may be observed, that the Diagonal Line being calculated only, for one Kind of Casks, cannot be true for any other, whose Dimensions and Curvature are not proportional.

The Construction of the Line of Areas.

This Line being diametrically applied to a Circle, shews the Area of that Circle by Inspection; and the Construction thereof will not seem difficult, if it be considered, that the Contents of all homogeneous Superficies, are in the same Proportion, as the Square of their Dimensions; that is, as the Area of any Circle, is to the Square of its Diameter; so is the Area of any other Circle, to the Square of its Diameter: Now the Square of the Diameter of that Circle, whose Area is one Ale

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Ale Gallon, is 359, and of that Circle whose Area is one Wine Gallon 294; therefore as 1 is to 359, or 294, so is the given Number of Ale, or Wine Gallons, to the Square of the Circle's Diameter; that is, multiply 359, or 294, by the Number of Gallons, the square Root of the Product will be the Diameter sought.

Let the Diameter of that Circle be required, whose Area is 4 Ale, or Wine Gallons.

$$\begin{array}{r}
 359 \\
 \times 4 \\
 \hline
 1436 \quad (37.89 \text{ Diameter of Ale Circle.} \\
 \times 9 \\
 \hline
 67) 536 \\
 \underline{469} \\
 748) 6700 \\
 \underline{5984} \\
 7569) 71600 \\
 \underline{68121} \\
 3479
 \end{array}$$

$$\begin{array}{r}
 294 \\
 \times 4 \\
 \hline
 1176 \quad (34.29 \text{ Diameter of Wine Circle} \\
 \times 9 \\
 \hline
 64) 276 \\
 \underline{256} \\
 682) 2000 \\
 \underline{1364} \\
 6849) 63600 \\
 \underline{61641} \\
 1959 \\
 P
 \end{array}$$

To reduce Beer Barrels to Ale Barrels.

Add an eighth Part of the Barrels and Firkins given, to the Barrels, Firkins and Gallons given ; the Sum is the Number of Barrels, Firkins and Gallons in Ale.

Let it be required to know how many Barrels, Firkins and Gallons of Ale, are contained in 34 B. 2 F. 7 G. of Beer.

	B.	F.	G.	
8)	34	2	7	
	4	1	2	Add.
	39	0	1	Ale.

Since 8 is contained 4 Times in 34, and 2 remains ; let 4 be set in the Place of Barrels, and the 2 which remains, being 2 Barrels, or 8 Firkins ; let this be added to the 2 Firkins, and the Sum will be 10 ; which being divided by 8, will give 1 ; this put in the Place of Firkins, and the remaining two in the Place of Gallons ; these Sums added together will produce 39 B. 0 F. 1 G. of Ale.

To reduce Ale Barrels to Beer Barrels.

Subtract the ninth Part of the Barrels and Firkins given, the Remainder will be the Number of Barrels, Firkins and Gallons in Beer.

Let it be required to reduce 39 B. 2 F. 3 G. of Ale, to Beer Measure.

	B.	F.	G.	
9)	39	2	3	
	4	1	5	Subtract.
	35	0	7	Beer.

To reduce Ale Gallons to Wine Gallons.

Set down the given Number of Gallons twice, one under the other, but the second Time one Place more to the Left-Hand; add these two together, and divide the Sum by 9, the Quotient will be the Quantity of Wine Gallons.

How many Gallons of Wine are equal to 124 Gallons of Ale?

$$\begin{array}{r}
 124 \\
 124 \\
 \hline
 9) 1364 \\
 \hline
 151.5 \text{ Wine Gallons.}
 \end{array}$$

To reduce Wine Gallons to Ale Gallons.

Multiply the given Number of Wine Gallons by 9, and divide the Product by 11, the Quotient will be the Quantity of Ale Gallons.

How many Gallons of Ale are equal to 108 Gallons of Wine?

$$\begin{array}{r}
 108 \\
 9 \\
 \hline
 11) 972 \\
 \hline
 88.3 \text{ Ale Gallons.}
 \end{array}$$

P A R T II.

Of the SLIDING-RULE.

This Instrument, whatsoever hath been said thereof, is the most ready and useful in the Art of Gauging, and in most Cases sufficiently exact: It

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was first invented to lay aside the Use of Compasses, which were very troublesome ; the former being much better adapted to perform the same Things by one Line sliding by another.

There have been, however, several Improvements made in this Instrument, and I flatter myself, that the expert Gauger will pardon me if I attempt one, which is in the Lines for finding the mean Diameter of Casks: Mr. Everard divides the Difference between the Frustums of two Cones, and the middle Frustum of a Spheroid, into three equal Parts, whereby in a Cask, whose greatest Content is not 130 Gallons, he makes the common Difference near 6 Gallons, which may cause a considerable Error. I have therefore, as is before noted, divided it into four equal Parts, rejecting the Frustums of two Cones, knowing it impracticable to make any such Cask ; and the Construction of those Lines I have communicated to Mr. John Gilbert in Postern-Row, Tower-Hill, by whom the Sliding-Rule, thus amended, is accurately made.

I shall first of all shew the Method of constructing the Lines on the Sliding-Rule, believing the Learner will thereby more readily attain the Knowledge of their Use.

Of the Construction of the Line D.

This Line is a single Line of Numbers, and is constructed from the Table of Logarithms, of which I have here transcribed so much, as is necessary for the Construction of this Line.

A T A B L E.

Num.	Log.	Num.	Log.
1.0	000	5.6	748
1.1	041	5.7	756
1.2	079	5.8	763
1.3	114	5.9	771
1.4	146	6.0	778
1.5	176	6.1	785
1.6	204	6.2	792
1.7	230	6.3	799
1.8	255	6.4	806
1.9	279	6.5	813
2.0	301	6.6	820
2.1	322	6.7	826
2.2	342	6.8	832
2.3	362	6.9	839
2.4	380	7.0	845
2.5	398	7.1	851
2.6	415	7.2	857
2.7	431	7.3	863
2.8	447	7.4	869
2.9	462	7.5	875
3.0	477	7.6	881
3.1	491	7.7	886
3.2	505	7.8	892
3.3	518	7.9	898
3.4	531	8.0	903
3.5	544	8.1	908
3.6	556	8.2	914
3.7	568	8.3	919
3.8	580	8.4	924
3.9	591	8.5	929
4.0	602	8.6	934
4.1	613	8.7	939
4.2	623	8.8	944
4.3	633	8.9	949
4.4	643	9.0	954
4.5	653	9.1	959
4.6	663	9.2	964
4.7	672	9.3	968
4.8	681	9.4	973
4.9	690	9.5	978
5.0	699	9.6	982
5.1	708	9.7	987
5.2	716	9.8	991
5.3	724	9.9	996
5.4	732	10.0	1000
5.5	740		

Now

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Now having drawn a Line exactly the Length of the Line D, divide it into 1000 equal Parts; then taking 301 of those Parts in the Compasses, set it off from the Beginning of the Line D, which will be the Distance for 2; again setting off 477 Parts from the Beginning of the Line D, there will be had the Place of 3, and so of the other Primes: Which being drawn, the intermediate Spaces may be set off in the like Manner, as 41 Parts, for 1.1; 79 Parts for 1.2; and 114 Parts for 1.3, &c.

Of the Lines A, B and C, on the Sliding-Rule.

These are only double Lines of Numbers; wherefore draw a Right Line equal to half the Length of the Lines A, B or C, and divide it into 1000 equal Parts, and set off the Distances as in the Line D.

Of the Line E on the Sliding-Rule.

The Line E is a triple Line of Numbers; wherefore draw a Right Line equal to a third Part of the Line E, and divide it into 1000 equal Parts, and set off the Distances by the Table as before.

Of the Line M D, or Malt Depth, on the Sliding-Rule.

This Line is only an inverted Line of Numbers, beginning at 2150 the Divisor for a Malt Bushel; whereby the Quantity of Malt in a Couch, or Floor, is found at one Operation; and a Sliding-Rule may be made to gauge Beer, or Wine, in Backs or Tuns at one Operation, if an inverted Line of Numbers was to begin at 282, or 231, the respective Divisors.

Of

Of the Lines of Varieties of Casks, for finding mean Diameters.

These are constructed from the Tables of Varieties, in the first Part of this Book, in this Manner: Admit it were required to know where 1 is to be set in the Line for a Spheroid. Look in the Table, and the nearest Number below it will be found .94, and the nearest Number above it 1.01: The Difference of these two Numbers is .07; and the Difference between .94 and 1.00 is .06; then say by the Rule of Proportion, As .07 is to .1; so is .06 to a fourth Number, viz. .085; this added to 1.4, the Inches and Parts corresponding to .94, the Sum is 1.485, against which on the Line of Inches stands 1 on the Spheroid Line: By proceeding in this Manner the following Table may be constructed.

A T A B L E.



	Spheroid.	I Variety.	II Variety.	III Variety.
	Inches.	Inches.	Inches.	Inches.
1	1.485	1.583	1.700	1.820
2	2.942	3.142	3.366	3.640
3	4.385	4.683	5.016	5.433
4	5.800	6.200	6.666	7.200
5	7.214	7.716	8.300	8.983
6	8.600	9.233	9.916	10.716
7	10.000	10.750		

The Line of Areas, which is on the Inside of the Sliders, is constructed as was taught in the first Part.

The Lines of Segments on the Sliding-Rule, are constructed in this Manner: Let there, first of all,

all, be made a spheroidal Cask, whose Content is 100 Gallons; then on a strait Rule measure the Bung Diameter for the Segments of a lying Cask, and the Length for the Segments of a standing Cask; the Bung Diameter or Length being divided into 1000 equal Parts; then pouring in one Gallon, observe how many Parts are cut on the Rule, and against those Parts on the Line of Numbers, set 1 on the Segment Line; so proceed for 2, 3, 4, 5 Gallons, &c. until the Cask is full, and the Line of Segments finished,

I shall now proceed to shew the Use of those Lines, so far as the Art of Gauging requires; and, first of all, how,

Any Number being proposed, to find the Point where such Number is represented on the Lines.

To make this appear more plain, I shall only describe the single Line of Numbers, which being well understood will give sufficient Insight to the rest.

This Line, which is marked D, is first of all divided into nine Parts, expressed by the Figures, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10; which for Distinction sake call Primes; these Primes are each of them divided again into ten Parts, which Parts call Seconds; the Seconds are each of them again divided, or supposed to be so, into ten Parts, called Thirds, and so *ad infinitum*. Now between the Primes 1 and 2, the Seconds are actually divided into ten Parts; but between 2 and 3, the Seconds are divided only into five Parts, and therefore each Part will signify two Thirds; and afterwards the Seconds are divided into two Parts only; so each Part will signify five Thirds.

The Learner having rightly considered this, it will be easy to express any given Number. Admit then, it were required to find the Point expressing 1715. Here 1 is a Prime, 7 the Seconds,

1 a Third, and 5 so many Fourths: Wherefore from 1 account seven of the largest Divisions, between 1 and 2, and that Point will express 17; then account one of the next greater Divisions, and that Point signifies 171: Now the Thirds not being divided, guess at the five Fourths, which must fall exactly in half the Division, and there making a Point, it will represent 1715, against which is W. G. the gauge Point for Wine Gallons: And in the same Manner may be found 1895, against A. G. the gauge Point for Ale Gallons; and here observe, that the same Point represents 1895, or 189.5, or 18.95, or 1.895, or .1895, or .01895, &c. So also the Prime which signifies 3 represents also 30, 300, 3000, &c.

But the less the Number of Figures, the more easy are they represented: Thus were it 26, from the Point 2 account 6 Seconds, which will be the representing Point.

The Practice of the following Problems, will much conduce to a farther Idea of the Divisions.

Having three given Numbers, to find a Fourth, in the same Proportion to the Third, as the Second is to the First.

Against the first Number on A, set the second Number on B; then against the third Number on A, is the fourth proportional Number on B.

Let the three given Numbers be 4, 6, and 8.

Against 4 on A, set 6 on B; then against 8 on A, is 12 on B; that is, as 4 is to 6; so is 8 to 12.

If 1 Pound of any Commodity costs 14 Pence, What will 6 Pounds of the same cost?

Against 1 on A, set 14 on B; then against 6 on A, is 84 on B; therefore the Price of 6 Pounds is 84 Pence.

And thus, as the Rule stands, one may see the Price of any Number of Pounds, viz. against 7
Q Pounds

Pounds 98 Pence; against 8 Pounds 112 Pence,
&c.

OF MULTIPLICATION.

The last Proposition being duly considered, Multiplication will be easy; for the Analogy will always be: As 1 is to the Multiplier; so is the Multiplicand to the Product.

Let it be required to multiply 7 into 9.

As 1 is to 7; so is 9, to 63.

Therefore against 1 on A, set one of the given Numbers on B; then against the other Number on A, is the Product on B.

Let it be required to multiply 16 by 14.

Set 1 on A, against 16 on B; then against 14 on A, is 224 on B.

Let it be required to multiply 43 by 24.

Set 1 on A, against 43 on B; then against 24 on A, is 1032 on B.

Now in order to discover how many Figures are in the Product, do thus: Compare the Value of the first Figures of the Product, with those of each of the given Numbers; if the first Figures of the Product be of less Value, than the first Figures of either of the given Numbers; there will be so many Figures in the Product, as are in both the Numbers given: But if the Value of the first Figures of the Product, be greater than the Value of the first Figures of either of the given Numbers; then there will be a Figure less in the Product, than are in the two given Numbers: Thus in the last Examples, the two first Figures in the Product 224, are found to be of greater Value, than either 14 or 16; therefore, there is one Figure less in the Product, than in both the given Numbers. But in the other Product 1032
the

the two first Figures are found to be of less Value, than either 43 or 24; wherefore, there is the same Number of Figures in the Product, as in both the given Numbers.

Multiplication of Decimals, differs not from *Multiplication of whole Numbers*; due Care being taken in pointing off the Decimals, as was shewn in the first Part.

When your Product hath more than three or four Figures, it is difficult to distinguish the latter Figures on the Rule; but in this Case it will be easy to bear one, or two in Memory: Thus, were it required to multiply 68 by 36, the Product will be 2448. For the last Figure is known to be an 8, by multiplying 6 by 8 mentally.

Of DIVISION.

In Division, the Analogy will always be: As the Divisor, is to 1; so is the Dividend, to the Quotient.

Wherefore against the Divisor on A, set one on B; then against the Dividend on A, is the Quotient on B.

Let it be required to divide 1032 by 24.

Set 1 on B, against 24 on A; then against 1032 on A, is 43 on B.

Let it be required to divide 15744 by 64.

Against 64 on A, set 1 on B; then against 15744 on A, is 246 on B.

N.B. If the Value of the first Figures of the Dividend, be less than those of the Divisor, there will be as many Figures in the Quotient, as there are in the Dividend more than are in the Divisor: But if the Value of the first Figures of the Dividend, be greater than those of the Divisor; then

the Number of Figures in the Quotient, will be one more than the Dividend contains, more than the Divisor.

Division of Decimals, is perform'd in the same Manner; due Regard being had to the Value of the Quotient, as was taught in the first Part: Thus were it required to divide .984 by 24. By the last Rule there must be two Figures in the Quotient, (*viz.* 41); but as there are three Decimals in the Dividend, more than are in the Divisor, there must be annex'd a Cypher on the Left-Hand, whence the Quotient becomes .041. Or thus; place your Divisor under your Dividend towards the Left-Hand, in such Sort that it may be subtracted; then the first Figure of the Quotient, will be of the same Value as that Figure in the Dividend, under which the Place of Units, in whole Numbers, stands in the Divisor.

Thus, in the last Example; setting the Divisor under the Dividend towards the Left-Hand, so that it may be subtracted, it will stand thus

$$\begin{array}{r} .984 \\ 24 \end{array}$$

Here the Place of Units 4 in the Divisor, stands under 8 the second Place of Decimals; wherefore the first Figure of the Quotient must stand in that Place, *viz.* .04.

Again, suppose that 2.768 was to be divided by .004, setting the Divisor, that it may be subtracted, it will stand thus,

$$\begin{array}{r} *. * 2.768 \\ *.004 \end{array}$$

Here filling up the Places with small Stars, to represent Figures, the Place of Units in the Divisor will be found to fall under the Place of Hundreds, in the Dividend; whence the Value of the first Figure in the Quotient, will be so many Hundreds, and consequently the Quotient will be 692.

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To find a Multiplier, to perform the Office of a Divisor.

Against the Divisor on A, set 1 on B; then against 1 on A, is the Multiplier on B.

Let it be required to find a Number, by which a given Number being multiplied, the Number produced shall be the same as if the given Number was divided by 125.

Against 125 on A, set 1 on B; then against 1 on A, is .008 on B.

To reduce a Vulgar Fraction, to a Decimal Fraction.

Against the Denominator on A, set 1 on B; then against the Numerator on A, is the Decimal Fraction on B.

Let it be required to reduce $\frac{3}{4}$ to a Decimal Fraction.

Against 24 on A, set 1 on B; then against 3 on A, is .125 on B.

To find the Value of a Decimal in known Parts of an Integer.

Against 1 on A, set the Decimal on B; then against the Number of Species, of the highest Denomination, contained in the Integer on A, is the Value on B.

Let it be required to know the Value of .775 of a Pound Sterling.

Against 1 on A, set .775 on B; then against 20, the Number of Shillings in a Pound on A, is 15.5 on B; that is, 15 Shillings and 5 Tenths.

Then against 1 on A, set .5 on B; then against 12, the Number of Pence in a Shilling on A, is 6 on B.

Thus the Value of the Decimal is found to be 15 Shillings and 6 Pence.

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To find a geometrical mean Proportional between two given Numbers.

Set one of the given Numbers on C, against the same Number on D; then against the other given Number on C, is the geometrical Mean on D.

Let the geometrical Mean between 8 and 18 be required.

Set 8 on C, against 8 on D; then against 18 on C, is 12 on D.

To Extract the square R O O T.

This is done by Inspection. First place 10 on C, against 10 on D; then against any Number on C, is its square Root on D: Observe if the Integers of the given Number be odd, you must seek the Number in the Left-Hand *Radius*; but if the Number of Integers be even, the given Number must be sought in the Right-Hand *Radius* of the Line C.

Let it be required to find the square Root of 441 and 64.

The first consisting of an odd Number of Figures, seek it in the Left-Hand *Radius* of the Line C; and against it on D, is 21 its Root.

The last consisting of an even Number of Figures, seek it in the Right-Hand *Radius* of the Line C; and against it on D, is 8 its Root.

To Extract the Cube R O O T.

Set 10 on D, against 1000 on E; then against the given Number on E, is its Cube Root on D: Observing if the Integers consist of 1, 4, 7 or 10 Figures, the given Number, is to be sought on the Left-Hand *Radius* of the Line E; if they consist of 2, 5, 8 or 11 Figures, they are to be sought in the middle *Radius*; and if they consist of

of 3, 6, 9 or 12 Figures, they are to be sought on the Right-Hand *Radius*.

Let it be required, to find the Cube Roots of 1728, 29791 and 140.608.

The first consisting of four Integers, seek it on the Left-Hand *Radius*; and against it on D, is 12 its Root.

The second consisting of five Integers, seek it on the middle *Radius*; and against it on D, is 31 its Root.

The last consisting of three Integers, seek it on the Right-Hand *Radius*; and against it on D, is 5.2 its Root.

Of a SUPERFICIES, and first of a CIRCLE.

The Diameter of a Circle being given; to find the Circumference.

Against 1 on A, set 3.1415 on B; then against any Diameter on A, is the Circumference on B.

Let the given Diameter be 30.

Against 1 on A, set 3.1415 on B; then against 30 on A, is 94.24 on B.

The Circumference of a Circle being given; to find the Diameter.

Set the Rule, as in the last Problem; then against the Circumference on B, is the Diameter on A.

Let the given Circumference be 43.98.

Set 3.1415 on B, against 1 on A; then against 43.98 on B, is 13.99 on A.

The Diameter of a Circle being given; to find the Side of a Square equal in Area to the given Circle.

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Against 1 on A, set .886 on B; then against the Diameter on A, is the Side of the Square on B.

Let it be required, to find the Side of a Square, the Content of which is equal to a Circle whose Diameter is 14.

Against 1 on A, set .886 on B; then against 14 on A, is 12.4 on B.

The Circumference of a Circle being given; to find the Side of a Square, the Area of which shall be equal to the said Circumference.

Against 1 on A, set .282 on B; then against the Circumference on A, is the Side of the Square on B.

Let the Circumference be 44.

Against 1 on A, set .282 on B; then against 44 on A, is 12.4 on B.

The Diameter of a Circle being given; to find the Side of the greatest Square, that can be inscrib'd therein.

Against 1 on A, set .707 on B; then against the Diameter on A, is the Side of the Square to be inscrib'd on B.

Let the given Diameter be 28.

Against 1 on A, set .707 on B; then against 28 on A, is 19.79 on B.

The Circumference of a Circle being given; to find the Side of the greatest Square, that can be inscrib'd therein.

Against 1 on A, set .225 on B; then against the Circumference on A, is the Side of the inscrib'd Square on B.

Let the given Circumference be 88.

Against

Against 1 on A, set .225 on B; then against 88 on A, is 19.8 on B.

The Diameter of a Circle being given; to find the superficial Content in Inches.

Against 1 on D, set .7854 on C; then against the Diameter on D, is the Content on C.

Let the given Diameter be 20.

Against 1 on D, set .7854 on C; then against 20 on D, is 314.16 on C.

The Diameter of a Circle being given; to find the Area in Ale, or Wine Gallons.

Set 1 on C, against the Gauge Point on D; then against the Diameter on D, is the Area on C.

Let the Diameter of the given Circle be 36.

Against A.G. on D, set 1 on C; then against 36 on D, is 3.61 Ale Gallons on C.

Against W.G. on D, set 1 on C; then against 36 on D is 44.0 Wine Gallons on C.

The Diameter of a Circle, or Cylinder, its whole Content, and Altitude of any Segment thereof being given; to find the Content of that Segment.

Parallel to the Line S.L. on the Sliding-Rule, is usually placed the Line S.C. which signifies the Segments of a Circle, or Cylinder; wherefore set the Diameter on Num. against 100 on S.C.; then against the Altitude on Num. is the Segment on S.C.

Then against 100 on A, set the whole Content on B; and against the Segment on A, is the Content of the Segment on B.

Let (*Fig. XXIII.*) represent the End of a Cylinder, whose Diameter is 30, the Wet Inches 6, and the whole Content 108 Gallons; the Quantity of Liquor in the Cylinder is required.

R

Set

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Set 30 on Num. against 100 on S.C.; then against 30 on Num. is 14 on S.C.

Against 100 on A, set 108 on B; then against 14 on A, is 15 Gallons on B.

The Transverse and Conjugate Diameters of an Ellipsis being given; to find its Area in Ale, or Wine Gallons.

Against 359 on A, set one Diameter on B; then against the other Diameter on A, is the Area in Ale Gallons on B.

Or against 294 on A, set one Diameter on B; then against the other Diameter on A, is the Area in Wine Gallons on B.

Let the $\left\{ \begin{array}{l} \text{Transverse} \\ \text{Conjugate} \end{array} \right\}$ Diameter be $\left\{ \begin{array}{l} 40. \\ 30. \end{array} \right.$

Against 359 on A, set 40 on B; then against 30 on A, is 3.34 Ale Gallons on B.

Against 294 on A, set 40 on B; then against 30 on A, is 4.08 Wine Gallons on B.

The Diameter and Depth of a circular Tun, whose Sides are perpendicular to its Base being given; to find its Content at one Operation.

Against the Gauge Point on D, set the Depth on C; then against the Diameter on D, is the Content on C.

Let the Diameter be 28.4, and Depth 16.5.

Against A.G. on D, set 16.5 on C; then against 28.4 on D, is 36.9 Ale Gallons on C.

Against W.G. on D, set 16.5 on C; then against 28.4 on D, is 45.2 Wine Gallons on C.

The Diameters and Depth of an elliptical Tun, whose Sides are perpendicular to the Base, being given; to find the Content.

First find a mean Diameter, between the Transverse and Conjugate; thus, Set

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Set one of the Diameters on C, against the same Diameter on D; then against the other Diameter on C, is the mean Diameter on D.

Then against the Gauge Point on D, set the Depth on C; and against the mean Diameter on D, is the Content on C.

Let the $\left\{ \begin{array}{l} \text{Transverse} \\ \text{Conjugate} \end{array} \right\}$ Diameter be $\left\{ \begin{array}{l} 40 \\ 30 \end{array} \right.$ Depth 24.

Set 30 on C, against 30 on D; and against 40 on C, is 34.64 the mean Diameter.

Against A.G. on D, set 24 on C; then against 34.64 on D, is 80.16 Ale Gallons on C.

Against W.G. on D, set 24 on C; then against 34.64 on D, is 97.92 Wine Gallons on C.

The Base and Perpendicular of a Triangle being given; to find the Area in Ale, or Wine Gallons.

Against 282 on A, set one Dimension on B; then against half the other Dimension on A, is the Area on Ale Gallons on B.

Against 231 on A, set one Dimension on B; then against half the other Dimension on A, is the Area in Wine Gallons on B.

Let the Base be 175, and Perpendicular 71.3.

Against 282 on A, set 71.3 on B; then against 87.5 on A, is 22.1 Ale Gallons on B.

Against 231 on A, set 71.3 on B; then against 87.5 on A, is 27.0 Wine Gallons on B.

The Side of a Square being given; to find the Diagonal.

Against 1 on A, set 1.414 on B; then against the Side on A, is the Diagonal on B.

Let the Side of the given Square be 41.

Against 1 on A, set 1.414 on B; and against 41 on A, 57.97 on B.

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The Difference between the Side and Diagonal of a Square being given; to find the Side.

Against 1 on A, set 2.414 on B; then against the given Difference on A, is the Side of the Square on B.

Let the given Difference be 24.

Against 1 on A, set 2.414 on B; then against 24 on A, is 57.9 on B.

The Side of a Square being given; to find its Area in Ale, or Wine Gallons.

Against 16.79 on D, set 1 on C; then against the Side of any Square on D, is the Area in Ale Gallons on C.

Against 15.19 on D, set 1 on C; then against the Side of any Square on D, is the Area in Wine Gallons on C.

Let the Side of the Square be 41.

Against 16.79 on D, set 1 on C; then against 41 on D, is 5.96 Ale Gallons on C.

Against 15.19 on D, set 1 on C; then against 41 on D, is 7.27 Wine Gallons on C.

The Side of a Square Tun, whose Sides are Perpendicular to its Bottom, and Depth being given; to find the Content in Ale, or Wine Gallons.

Against 16.79 on D, set the Depth on C; then against the Side on D, is the Content in Ale Gallons on C.

Against 15.19 on D, set the Depth on C; then against the Side on D, is the Content in Wine Gallons on C.

Let the Side be 46, and Depth 26.

Against 16.79 on D, set 26 on C; then against 46 on D, is 194.8 Ale Gallons on C.

Against 15.19 on D, set 26 on C; then against 46 on D, is 237.9 Wine Gallons on C.

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The Length and Breadth of an Oblong being given ; to find the Area in Ale, or Wine Gallons.

Against 282 on A, set one Dimension on B ; then against the other Dimension on A, is the Area in Ale Gallons on B.

Against 231 on A, set one Dimension on B ; then against the other Dimension on A, is the Area in Wine Gallons on B.

Let the Length of the given Oblong be 201, and Breadth 82.

Against 282 on A, set 201 on B ; then against 82 on A, is 58.4 Ale Gallons on B.

Against 231 on A, set 201 on B ; then against 82 on A, is 71.2 Wine Gallons on B.

The Side and Perpendicular of a Rhombus being given ; to find its Area in Ale, or Wine Gallons.

Against 282 on A, set the Side on B ; then against the Perpendicular on A, is the Area in Ale Gallons on B.

Against 231 on A, set the Side on B ; then against the Perpendicular on A, is the Area in Wine Gallons on B.

Let the Side of the given Rhombus be 121, and Perpendicular 109.

Against 282 on A, set 121 on B ; then against 109 on A, is 46.7 Ale Gallons on B.

Against 231 on A, set 121 on B ; then against 109 on A, is 57.0 Wine Gallons on B.

The Length and Perpendicular of a Rhomboides being given ; to find its Area in Ale, or Wine Gallons.

Against 282 on A, set the Length on B ; then against the Perpendicular on A, is the Area in Ale Gallons on B.

Against 231 on A, set the Length on B ; then against the Perpendicular on A, is the Area in Wine Gallons on B.

Let

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Let the Length of the given Rhomboides be 185, and Perpendicular 92.

Against 282 on A, set 185 on B; then against 92 on A, is 60.3 Ale Gallons on B.

Against 231 on A, set 185 on B; then against 92 on A, is 73.6 Wine Gallons on B.

The Diagonal and Perpendiculars of a Trapezium being given; to find its Area in Ale, or Wine Gallons.

Against 282 on A, set the Diagonal on B; then against half the Sum of the Perpendiculars on A, is the Area in Ale Gallons on B.

Against 231 on A, set the Diagonal on B; then against half the Sum of the Perpendiculars on A, is the Area in Wine Gallons on B.

Or thus: Against 564 on A for Ale Gallons, or 462 on A for Wine Gallons, set the Diagonal on B; then against the Sum of the Perpendiculars on A, is the Area on B.

Let the Diagonal be 206, and Sum of the Perpendiculars 147.

Against 564 on A, set 206 on B; then against 147 on A, is 53.69 Ale Gallons on B.

Against 462 on A, set 206 on B; then against 147 on A, is 65.54 Wine Gallons on B.

The Perpendicular and Sum of the Sides, of any regular Polygon being given; to find its Area in Ale, or Wine Gallons.

Against 564 on A, set the Perpendicular on B; then against the Sum of the Sides on A, is the Area in Ale Gallons on B.

Against 462 on A, set the Perpendicular on B; then against the Sum of the Sides on A, is the Area in Wine Gallons on B.

Let there be a Polygon of 5 Sides, whose Side is 50, and Perpendicular 34.4.

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Against 564 on A, set 34.4 on B; then against 250 on A, is 15.24 Ale Gallons on B.

Against 462 on A, set 34.4 on B; then against 250 on A, is 18.61 Wine Gallons on B.

The Side only of a regular Polygon being given; to find the Area in Ale, or Wine Gallons.

Num. of Sides.	Ale Gallons.	Wine Gall.
Five	.006100	.007447
Six	.009212	.011245
Seven	.012888	.015733
Eight	.017120	.020899
Nine	.021921	.026760
Ten	.027280	.033302
Eleven	.033210	.040541
Twelve	.039700	.048464

Against 1 on D, set the tabular Number on C; then against the Side on D, is the Area on C.

Let a Polygon of 9 Sides be given, whose Side is 40.

Against nine Sides, the tabular Number for Ale is .021921, and for Wine is .026760.

Against 1 on D, set .021921 on C; then against 40 on D, is 35.07 Ale Gallons on C.

Against 1 on D, set .026760 on C; then against 40 on D, is 42.81 Wine Gallons on C.

The Area of the Base, of any upright Vessel, and Depth of Liquor being given; to find the Content.

Against 1 on A, set the Area on B; then against the Depth on A, is the Content on B.

Let the Area of a square Vessel be 5.96 Ale Gallons, or 7.27 Wine Gallons, and Depth of the Liquor 27.

Against 1 on A, set 5.96 on B; then against 27 on A, is 160.9 Ale Gallons on B.

Against

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Against 1 on A, set 7.27 on B; then against 27 on A, is 196.2 Wine Gallons on B.

The Diameter of the Base and Altitude of a Cone being given; to find its Content in Ale, or Wine Gallons.

Against the Gauge Point on D, set one Third of the Altitude on C; then against the Diameter of the Base on D, is the Content on C.

Let the given Diameter be 28.8, and Altitude 30.

Against A.G. on D, set 10 on C; then against 28.8 on D, is 23.1 Ale Gallons on C.

Against W.G. on D, set 10 on C; then against 28.8 on D, is 28.2 Wine Gallons on C.

The Side of the Base and Altitude of a square Pyramid being given; to find the Content in Ale, or Wine Gallons.

Against 29.08 on D, set the Altitude on C; then against the Side on D, is the Content in Ale Gallons on C.

Against 26.32 on D, set the Altitude on C; then against the Side on D, is the Content in Wine Gallons on C.

Let the Side of the Base be 48, and Altitude 68.

Against 29.08 on D, set 68 on C; then against 48 on D, is 185.1 Ale Gallons on C.

Against 26.32 on D, set 68 on C; then against 48 on D, is 225.9 Wine Gallons on C.

To find the Content of any Pyramid in Ale, or Wine Gallons.

Find the Area of the Base by one of the foregoing Rules; then against 3 on A, set the Area on B; and against the Altitude on A, is the Content on B.

Let

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Let the Area of the Base be 15.24 Ale Gallons, or 18.61 Wine Gallons, and Depth 63.

Against 3 on A, set 15.24 on B; then against 63 on A, is 320.0 Ale Gallons on B.

Against 3 on A, set 18.61 on B; then against 63 on A, is 390.8 Wine Gallons on B.

To find the Content of the Frustum of a Cone, in Ale, or Wine Gallons.

Against the Gauge Point on D, set the Depth on C; then against the half Sum of the top and bottom Diameters on D, is a fourth Number on C to be reserved.

Against the Gauge Point on D, set one Third of the Depth on C; then against half the Difference of the Diameters on D, is a Number on C; which, added to the Number before reserved, gives the whole Content.

Let there be the Frustum of a Cone, whose top Diameter is 24.6, and bottom Diameter 16.4, and Depth 36.

24.6	24.6
16.4	16.4
-----	-----
2) 41.0	2) 8.2
-----	-----

20.5 Half Sum of Diam.

4.1 Half Difference of the Diam.

Against A. G. on D, set 36 on C; then against 20.5 on D, is 42.13 Ale Gallons on C,
Against A. G. on D, set 12 on C; then against 4.1 on D, is 0.56 on C.

42.69 Content in Ale Gall.

Against W. G. on D, set 36 on C; then against 20.5 on D, is 51.44 Wine Gallons on C,
Against W. G. on D, set 12 on C; then against 4.1 on D, is 0.68 on C.

52.12 Cont. in Wine Gall.

To find a Diameter in any Part of the Frustum of a Cone.

Against the whole Depth on A, set the Difference of top and bottom Diameters on B; then
S
against

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against the Distance of the Diameter required, from the lesser End on A, is a fourth Number on B; which, added to the lesser Diameter, the Sum will be the Diameter sought.

Let the Frustrum be the same as in the last Proposition, and the Diameter required at 10 Inches from the Bottom.

Against 36 on A, set 8.2 on B; then against 10 on A, is 2.27 on B. Which added to 16.4 the Sum is 18.67; the Diameter at 10 Inches from the Bottom.

The same Method may be us'd, to find the Dimensions in any Part of the Frustrum of any Pyramid.

To find the Content of the Frustrum of a square Pyramid in Ale, or Wine Gallons.

Against 16.79 on D for Ale, or 15.19 on D for Wine, set the Depth on C; then against half the Sum of the top and bottom Sides on D, is a fourth Number on C, to be reserved. Then against 16.79 on D for Ale, or 15.19 on D for Wine, set one Third of the Depth on C; then against half the Difference of the top and bottom Sides on D, is a fourth Number on C; which, added to the Number before reserved, the Sum will be the Content.

Let the Side of the Top be 24.6, of the Bottom 16.4, and the Depth 36.

$$\begin{array}{r} 24.6 \\ 16.4 \\ \hline \end{array}$$

$$\begin{array}{r} 2) \ 41.0 \\ \hline \end{array}$$

20.5 Half Sum of Sides.

$$\begin{array}{r} 24.6 \\ 16.4 \\ \hline \end{array}$$

$$\begin{array}{r} 2) \ 8.2 \\ \hline \end{array}$$

4.1 Half Difference of the Sides.

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Against 16.79 on D, set 36 on C; then against 20.5 on D, is 53.64 Ale Gallons on C.

Against 16.79 on D, set 12 on C; then against 4.1 on D, is 0.71 on C; which, added to 53.64, gives 54.36 Ale Gallons for the Content.

Against 15.19 on D, set 36 on C; then against 20.5 on D, is 65.47 Wine Gallons on C.

Against 15.19 on D, set 12 on C; then against 4.1 on D, is 0.87 on C; which, being added to 65.47, gives 66.34 Wine Gallons for the Content.

To find the Content of the Frustrum of any Pyramid in Ale, or Wine Gallons.

Find the Area of the Top and Bottom, and a geometrical Mean between those Areas; add these three together; then against 3 on A, set the Depth on B; and against the Sum of the Areas on A, is the Content on B.

Let the Frustrum be the same as in the last Proposition.

By the Rule for finding the Area of a Square, the Area of the Top is found to be 2.15, and that of the Bottom 0.95 Ale Gallons.

Whence setting 0.95 on C, against 0.95 on D; then against 2.15 on C, is 1.43 on D, the geometrical mean Area: These added together as follows,

$$\begin{array}{r} 0.95 \\ 2.15 \\ 1.43 \\ \hline \end{array}$$

The Sum will be 4.53

Against 3 on A, set 36 on B; then against 4.53 on A, is 54.3 Ale Gallons on B.

If the Content in Wine was required, the Area of the Top is 2.63, of the Bottom is 1.16.

S 2

Therefore

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Therefore set 1.16 on C, against 1.16 on D; then against 2.63 on C, is 1.4 on D, the geometrical mean Area: These three added together as follows,

$$\begin{array}{r} 1.16 \\ 2.63 \\ 1.74 \\ \hline \end{array}$$

The Sum will be 5.53

Against 3 on A, set 36 on B; then against 5.53 on A, is 66.3 Wine Gallons on B.

To find the Content of a Sphere in Ale, or Wine Gallons.

Against 23.2 on D, set the Diameter on C; then against the Diameter on D, is the Content in Ale Gallons on C.

Against 21 on D, set the Diameter on C; then against the Diameter on D, is the Content in Wine Gallons on C.

Let the Diameter of the given Sphere be 30.

Against 23.2 on D, set 30 on C; then against 30 on D, is 50.1 Ale Gallons on C.

Against 21 on D, set 30 on C; then against 30 on D, is 61.1 Wine Gallons on C.

Or thus: Against 1 on D, set .001856 on E; then against the Diameter on D, is the Ale Gallons on E.

Against 1 on D, set .002266 on E; then against the Diameter on D, is the Wine Gallons on E.

Therefore, against 1 on D, set .001856 on E; then against 30 on D, is 50.1 Ale Gallons on E.

Against 1 on D, set .002266 on E; then against 30 on D, is 61.1 Wine Gallons on E.

The Height, and Diameter of the Base of a spherical Segment being given; to find the Sphere's Diameter.

Against

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Against the Height on D, set the Height on C; then against half the Diameter of the Base on D, is the Remainder of the Sphere's Axis on C.

This Remainder, added to the Segment's Height, gives the Sphere's Diameter.

Let there be a spherical Segment, whose Height is 6, and Diameter of its Base 24.

Against 6 on D, set 6 on C; then against 12 on D, is 24 on C.

To 24 add 6, the Segment's Height; the Sum will be 30, the Sphere's Diameter.

The Diameter and Height of a spherical Segment being given; to find the Content in Ale, or Wine Gallons.

Against the Gauge Point on D, set double the Height on C; then against half the Diameter on D, is a fourth Number on C, to be reserved.

Against the Gauge Point on D, set two Thirds of the Height on C; and against the Height on D, is a fourth Number on C; which, added to the Number before reserved, the Sum will be the Content.

Let there be a spherical Segment, whose Diameter is 28, and Height 8.

Against A. G. on D, set 16 on C; then against 14 on D, is 8.73 Ale Gallons on C.
Against A. G. on D, set 5.33 on C; then against 8 on D, is 0.95 on C.

9.68 = Cont. in Ale Gall.

Against W. G. on D, set 16 on C; then against 14 on D, is 10.66 Wine Gallons on C.
Against W. G. on D, set 5.33 on C; then against 8 on D, is 1.15 on C.

11.81 = Cont. in W. Gall.

The Diameter of the whole Sphere, and Height of the Segment being given; to find the Content in Ale, or Wine Gallons.

From twice the Diameter of the Sphere, subtract once and a Third the Height of the Segment, and note the Remainder.

Against

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Against the Gauge Point on D, set the Remainder on C; then against the Height on D, is the Content on C.

Let the Diameter of the Sphere be 32.5, and the Height of the Segment 8.

32.5	3) 8.00
32.5	———
———	2.66 Add
65.0	———
10.66	10.66
———	
54.34	

Against A. G. on D, set 54.34 on C; then against 8 on D, is 9.68 Ale Gallons on C.

Against W. G. on D, set 54.34 on C; then against 8 on D, is 11.81 Wine Gallons on C.

Of CASK GAUGING.

On one Side of the Sliding-Rule, is a Line of Inches divided into Tenths; and parallel thereto, are the Lines of Varieties, for finding the mean Diameters of Casks.

To find the mean Diameter.

Seek the Difference of the Head and Bung Diameters, on the Line of Inches; and against it on the Line representing the Variety of the Cask, is a Number; which, added to the Head Diameter, the Sum will be the mean Diameter.

To find the Content in Ale, or Wine Gallons.

Against the Gauge Point on D, set the Length of the Cask on C; then against the mean Diameter on D, is the Content on C.

Let

Let there be a spheroidal Cask, whose head Diameter is 26.4, the Bung Diameter is 32.6, and Length 48.5; the Content of which is required.

$$\begin{array}{r} 32.6 \\ 26.4 \\ \hline \end{array}$$

6.2 Difference of Diameters.

Seek 6.2 on the Line of Inches, and against it on the Line mark'd Spheroid is 4.28; which, added to 26.4 the head Diameter, the Sum is 30.68, the mean Diameter.

Against A.G. on D, set 48.5 on C; then against 30.68 on D, is 127.1 Ale Gallons on C.

Against W.G. on D, set 48.5 on C; then against 30.68 on D, is 155.2 Wine Gallons on C.

Let it be required to find the Content of the former Cask; admit it to be of the first Variety.

Seek 6.2 on the Line of Inches, and against it on the Line of the first Variety, is found 4.0; which, being added to 26.4 the head Diameter, the Sum is 30.4 the mean Diameter.

Against A.G. on D, set 48.5 on C; then against 30.4 on D, is 124.8 Ale Gallons on C.

Against W.G. on D, set 48.5 on C; then against 30.4 on D, is 152.3 Wine Gallons on C.

Let the same Cask be supposed of the second Variety; and let the Content be required in Ale, or Wine Gallons.

Against 6.2, the Difference of Diameters on the Line of Inches, will be found 3.72, on the Line of the second Variety; which, added to 26.4 the head Diameter, the Sum is 30.12 the mean Diameter.

Against

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Against A.G. on D, set 48.5 on C; then against 30.12 on D, is 122.5 Ale Gallons on C.

Against W.G. on D, set 48.5 on C; then against 30.12 on D, is 149.5 Wine Gallons on C.

Again, Let the same Cask be supposed of the third Variety; and let the Content be required in Ale, or Wine Gallons.

Against 6.2, the Difference of Diameters, on the Line of Inches, will be found 3.43 on the Line of the third Variety; which, being added to 26.4, the Sum will be 29.83 the mean Diameter.

Against A.G. on D, set 48.5 on C; then against 29.83 on D, is 120.2 Ale Gallons on C.

Against W.G. on D, set 48.5 on C; then against 29.83 on D, is 146.7 Wine Gallons on C.

Of Ullaging Casks, or finding the Quantity of Liquor in a Cask partly full.

On the Side of the Rule are two Lines of Segments; the one marked S.L. signifying Segments lying; the other marked S.S. signifying Segments standing: Between these two Lines, slides a Line of Numbers, marked Num. divided in the same Manner as the Lines A and B.

To find the Quantity of Liquor in a lying Cask.

Having by the former Rules, found the whole Content in Ale or Wine Gallons, proceed in this Manner: Set the Bung Diameter on the Line of Numbers, against 100 on S.L; then against the Wet Inches on the Line of Numbers, is a fourth Number on S.L, called the Segment: Then if against 100 on A, be set the whole Content on B; against the Segment on A, will be the Quantity in the Cask on B.

Let there be a Cask, whose whole Content is 108.8 Ale Gallons, or 132.8 Wine Gallons; the Bung Diameter 31, and Wet Inches 9. The Quantity of

of Ale, or Wine in this Cask, is required to be known.

Against 31 on Num. set 100 on S.L; then against 9 on Num. is 22.2 on S.L.

Against 100 on A, set 108.8 on B; then against 22.2 on A, is 24.1 Ale Gallons on B.

Against 100 on A, set 132.8 on B; then against 22.2 on A, is 29.4 Wine Gallons on B.

Here the Quantity of Liquor, found by the Line of Segments, is in Ale Gallons .4 too little, and in Wine Gallons near .5 too little: The Occasion of this Error is, that the Lines of Segments on the Rule, were constructed for a Cask, whose Bung and Head Diameters, had a greater Proportion to the Length, than the Cask here proposed. And for this Reason all Casks, whose Lengths have a greater Proportion to the Diameters, than the Cask, for which this Line was constructed, will gauge somewhat too little, when not half full, and somewhat too much, when above half full.

This Error, however, is for the most Part inconsiderable; but if a greater Approximation be required, make use of the following Method; which, by repeated Trials, I have always found to approach the Truth very nearly in all kind of Casks.

Find, by the Instructions before given, the mean Diameter of the Cask; and subtract it from the Bung Diameter, and half the Remainder subtract from the Wet Inches, taken at the Bung; the Remainder is the Wet Inches, at the mean Diameter; which call the mean Wet Inches.

Now for the following Operation, I have placed a Line of Segments of a Circle mark'd S.C. parallel to the Line S.L. on one Side of the Sliding Rule.

Set the mean Diameter on Num. against 100 on S.C; then against the mean Wet Inches on Num. is the Segment on S.C.

T

Then

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Then against 100 on A, set the Content on B; and against the Segment on A, is the Quantity in the Cask on B.

Now let there be a Cask, whose Diameter at the Head is 25, at the Bung 31, and the Length 46: This Cask taken as the middle Frustum of a Spheroid, the Content will be 108.8 Ale Gallons, or 132.8 Wine Gallons, and the mean Diameter 29.14; which subtracted from 31, the Bung Diameter, the Remainder is 1.86, the half of which .93, subtracted from 9 the Wet Inches, the Remainder will be 8.07 the mean Wet Inches.

Set 29.14 on Num. against 100 on S.C; then against 8.07 on Num. is 22.5 on S.C.

Against 100 on A, set 108.8 on B; then against 22.5 on A, is 24.4 Ale Gallons on B.

Against 100 on A, set 132.8 on B; then against 22.5 on A, is 29.9 Wine Gallons on B.

To find the Quantity of Liquor in a standing Cask.

Having found the whole Content of the Cask, proceed thus: Set the Length on Num. against 100 on S.S; then against the Wet Inches on Num. is the Segment on S.S.

Against 100 on A, set the Content on B; then against the Segment on A, is the Quantity of Liquor in the Cask on B.

Let there be a Cask, whose whole Content is 108.8 Ale Gallons, or 132.8 Wine Gallons, the Length 46, and the Wet Inches 37; the Quantity of Liquor in the Cask is required.

Set 46 on Num. against 100 on S.S; then against 37 on Num. is 82.7 on S.S.

Against 100 on A, set 108.8 on B; then against 82.7 on A, is 89.9 Ale Gallons on B.

Against 100 on A, set 132.8 on B; then against 82.7 on A, is 109.8 Wine Gallons on B.

Of

Of the Line M. D. or Malt Depth.

This Line, as is before observed, is an inverted Line of Numbers, and is used for the ready finding, how many Malt Bushels are contained in a Couch, or Floor of Malt: It begins at 2150, the Divisor for a Malt Bushel, and shews the Content of all Prisms at one Operation, whose Bases require but two Dimensions, for the Investigation of their Areas. The Operation will be always thus: Set one of the Dimensions on B, against the Depth on M. D.; then against the other Dimension on A, is the Content in Malt Bushels on B.

Let there be a Couch or Floor of Malt, whose Length is 230, the Breadth 180, and the Depth 3.6; how many Malt Bushels doth this Floor contain?

Set 180 on B, against 3.6 on M. D.; then against 230 on A, is 69.3 Malt Bushels on B.

Let there be a Triangular Floor, whose Base is 230, and Perpendicular 180, and Depth 4.8; how many Malt Bushels doth it contain? Half of the Base is 115: Therefore set 180 on B, against 4.8 on M. D.; then against 115 on A, is 46.2 Malt Bushels on B.

Let there be a Couch in the Form of a Rhombus, whose Side is 236, Perpendicular 150, and Depth 13.4; how many Malt Bushels does it contain? Set 236 on B, against 13.4 on M. D.; then against 150 on A, is 220.6 Malt Bushels on B.

Let there be a Rhomboidal Floor of Malt, whose Length is 236, Perpendicular 75, and Depth 6.7; how many Malt Bushels doth it contain? Set 236 on B, against 6.7 on M. D.; then against 75 on A, is 55.1 Malt Bushels on B.

Let there be a Floor, whose Form is a Trapezium, the Base 206, half the Sum of the Perpendiculars 73.5; and Depth 3.5; how many Malt Bushels are contain'd in this Floor? Set 206 on B, against 3.5 on M.D.; then against 73.5 on A, is 24.6 Malt Bushels on B.

Let there be a Floor of Malt in the Form of a regular Polygon, half the Sum of whose Sides is 125, the Perpendicular is 34.4, and Depth is 6.3; how many Malt Bushels are contained therein? Set 125 on B, against 6.3 on M.D.; then against 34.4 on A, is 126 Malt Bushels on B.

To find the Content of a Cylinder in Malt Bushels.

There are on the Line D, two Gauge Points for Malt, mark'd M. S. and M. R. signifying Malt Square, and Malt Round; the first of these is in a great Measure useless, all Operations belonging to it, being more readily performed by the Line M. D.: However both their Uses are shewn, and first of the Line M. R.

Let there be a Cylinder, whose Diameter is 46.4, and Depth 24.6; how many Malt Bushels are contained therein? Set the Gauge Point on D, against the Depth on C; then against the Diameter on D, is the Content in Malt Bushels on C.

Set M. R. on D, against 24.6 on C; then against 46.4 on D, is 19.3 Malt Bushels on C.

To find the Content of an upright elliptical Vessel, in Malt Bushels.

First find a geometrical Mean between the Transverse and Conjugate Diameters, as is before taught in the first Part; then against M. R. on D, set the Depth on C; and against the mean Diameter on D, is the Content in Malt Bushels on C.

Let

Let there be an elliptical Vessel, whose Transverse Diameter is 40, the Conjugate Diameter 30, and Depth 25.5; how many Malt Bushels are contained in this Vessel? Set 30 on C, against 30 on D; then against 40 on C, is 34.64 on D, the mean Diameter.

Against M. R. on D, set 25.5 on C; then against 34.64 on D, is 11.1 Malt Bushels on C.

Of the Gauge Point M. S.

In using this Point, two Operations are always required, unless it be in a Square. The first Operation is to find the Side of a Square, whose Area is equal to the Base of the given Utensil; and in right-lined Bases, whose Areas are found by two Dimensions, is done thus: Set one Dimension on C, against the same Dimension on D; then against the other Dimension on C, is the Side of the Square equal thereto on D.

Then against M. S. on D, set the Depth on C; and against the Side of the Square on D, are the Malt Bushels on C.

Let the Base of the given Utensil be in the Form of an Oblong, whose Length is 230, Breadth 180, and Depth 3.6.

Against 230 on C, set 230 on D; then against 180 on C, is 203.46 on D.

Against M. S. on D, set 3.6 on C; then against 203.46 on D, is 69.3 Malt Bushels on C.

Again, Let there be a triangular Couch, whose Base is 230, and half the Perpendicular 90, and Depth 4.8.

Against 230 on C, set 230 on D; then against 90 on C, is 143.8 on D.

Against M. S. on D, set 4.8 on C; then against 143.8 on D, is 46.2 Malt Bushels on C.

The

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The Method of finding the Areas of Figures, and of inching Utensils, is the same in Malt, as in Ale, except in the Divisors, which are as follow,

In right lined Figures, } the Divisor for M. G. is { 268.8 } for M.B. { 2150.
In the Circle and Ellipsis, } { 342.2 } { 2738.

There is also on the Line A, a Gauge Point for Malt, mark'd M.B. at 2150, signifying Malt Bushels: The Use of this, is to find the Area of right-lined Figures, in Malt Bushels; and is done thus:

Against M.B. on A, set one Dimension on R; then against the other Dimension on A, is the Area in Malt Bushels on B.

Admit an Oblong, whose Length is 430, and Breadth 107.5; How many Malt Bushels does it contain, at one Inch deep?

Against M.B. on A, set 430 on B; then against 107.5 on A, is 21.5 Malt Bushels on B.

Useful PROBLEMS.

The Content and Diameter of a Cylinder being given; to find the Depth.

Set the Diameter on D, against the Content on C; then against the Gauge Point on D, is the Length on C.

Let a Cylinder be given, whose Content is 36.9 Ale Gallons, and Diameter is 28.4; and let the Depth be required.

Set 28.4 on D, against 36.9 on C; then against A. G. on D, is 16.5 on C, the Depth.

The Content and Depth of a Cylinder being given; to find the Diameter.

Against the Depth on C, set the Gauge Point on D; then against the Content on C, is the Diameter on D.

Let a Cylinder be given, whose Content is 45.2 Wine Gallons, and Depth 16.5; the Diameter is required.

Against 16.5 on C, set W.G. on D; then against 45.2 on C, is 28.4 on D, the Diameter.

The Content of a Cylinder, and Proportion the Diameter hath to the Depth being given; to find the Diameter and Depth.

Set the Gauge Point on D, against the Proportion of the Depth on C; then against the Proportion of the Diameter on D, is a fourth Number on C.

Against this fourth Number on E, set the Proportion of the Diameter, or Depth on D; then against the given Content on E, is the Diameter, or Depth on D.

Let a Cylinder be given, whose Content is 87 Ale Gallons, and the Diameter is in Proportion to the Depth, as 10 is to 20; the Diameter and Depth are required.

Set A. G. on D, against 20 on C; then against 10 on D, is 5.57 on C, the fourth Number.

Against 5.57 on E, set 10 on D; then against 87 on E, is 25 on D, the Diameter.

Against 5.57 on E, set 20 on D; then against 87 on E, is 50 on D, the Depth.

The Content, Depth, and Difference of the Diameters of a conical Tun being given; to find the Diameters.

Set the Gauge Point on D, against one Third of the Depth on C; then against half the Difference of Diameters on D, is a fourth Number on C; which Number is to be subtracted from the given Content, and the Remainder reserved.

Then set the Gauge Point on D, against the Depth on C; and against the Remainder before reserved

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reserved on C, is half the Sum of the Diameters on D.

To, and from this half Sum, add and subtract half the Difference of Diameters; the Sum will be the greater Diameter, and the Difference the lesser Diameter.

Let there be a conical Tun, whose Content is 170 Ale Gallons, the Depth 36, and Difference of the Diameters 8.4; the Diameters are required.

Half the Difference of Diameters is 4.2.

One Third of the Depth is 12.0.

Set A. G. on D, against 12 on C; then against 4.2 on D, is .59 on C.

From	170
Subtract	.59.

Remainder 169.41.

Set A. G. on D, against 36 on C; then against 169.41 on C, is 41.1 on D.

	41.1		41.1
Add	4.2	Subt.	4.2

Greater Diameter 45.3 36.9 Lesser Diamet.

The Content, Depth, and Proportion of the Diameters of a conical Tun being given; to find the Diameters.

Find, by the Rules before given, the Content of a conical Tun, whose Diameters and Depth are the same, as the Proportions of the Diameters and Depth here given; then against the Content thus found on C, set either of the given Proportions on D; then against the given Content on C, is the corresponding Diameter on D.

Let

Let there be a conical Tun, whose Content is 84 Ale Gallons; the Depth 36, and the Diameters in proportion as 6 to 9; the Diameters are required.

Now the Content of a Frustum of a Cone, whose lesser Diameter is 6, greater Diameter 9, and Depth 36, will be found to be 5.71 Ale Gallons.

Against 5.71 on C, set 6 on D; then against 84 on C, is 23 on D, the lesser Diameter.

Against 5.71 on C, set 9 on D; then against 84 on C, is 34.5 on D, the greater Diameter.

The Content and the Proportions of the Diameters, and Depth, of a conical Tun being given; to find the Diameters and Depth.

With the given Proportions, find the Content of a conical Frustum, as if the said Proportions were real Dimensions.

Then against the Content thus found on E, set either of the given Proportions on D; then against the given Content on E, is the corresponding Dimension on D.

Let there be a conical Tun, whose Content is 60.3 Ale Gallons, and the Diameters and Depth are in Proportion as 6, 7 and 8; the Diameters and Depth are required.

Now these Proportions being used, as the real Dimensions of a conical Tun, would produce the Content .939.

Against .939 on E, set 6 on D; then against 60.3 on E, is 24 on D, the lesser Diameter.

Against .939 on E, set 7 on D; then against 60.3 on E, is 28 on D, the greater Diameter.

Against .939 on E, set 8 on D; then against 60.3 on E, is 32 on D, the Depth.

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The Areas of two like Superficies being given, and the Dimensions of one of them; to find the Dimensions of the other.

Against the Area of any Superficies on C, set one of its Dimensions on D; then against any other Area on C, is the corresponding Dimension on D.

Let an Ellipsis be given, whose Transverse Diameter is 40, Conjugate Diameter is 30, and Area is 3.34 Ale Gallons; it is required to find the Transverse and Conjugate Diameters of a like Ellipsis, whose Area shall be 7.52 Ale Gallons.

Against 3.34 on C, set 40 on D; then against 7.52 on C, is 60 on D, the transverse Diameter.

Against 3.34 on C, set 30 on D; then against 7.52 on C, is 45 on D, the Conjugate Diameter.

The Contents of two like Solids, and the Dimensions of one of them being given; to find the Dimensions of the other.

Against the Content of any Solid on E, set one of its Dimensions on D; then against the Content of any like Solid on E, is the corresponding Dimension on D.

Let there be a Cask, whose Head Diameter is 20, Bung Diameter is 23.4, Length 28, and the Content in Ale Gallons 38.8; and let it be required to find the Dimensions of a like Cask, whose Content is 131 Ale Gallons.

Against 38.8 on E, set 20 on D; then against 131 on E, is 30 on D, the Head Diameter.

Against 38.8 on E, set 23.4 on D; then against 131 on E, is 35.1 on D, the Bung Diameter.

Against 38.8 on E, set 28 on D; then against 131 on E, is 42 on D, the Length.

The Content, Length, Difference of the Diameters, and Variety of a Cask being given; to find the Diameters.

Against the Length on C, set the Gauge Point on D; then against the Content on C, is the Mean Diameter on D.

Then seek the Difference of the Diameters on the Line of Inches, and take out the Number opposite thereto on the Line of the Variety of the Cask; this Number subtracted from the Mean Diameter, leaves the Head Diameter.

Let the given Cask be the Frustrum of a Spheroid, the Content 108.5 Ale Gallons, Length 48 and Difference of the Diameters 5.1; the Head and Bung Diameters are required.

Against 48 on C, set A. G. on D; then against 108.5 on C, is 28.5 on D, the Mean Diameter.

Against 5.1 on the Line of Inches, is 3.5 on the Spheroid Line.

From 28.5	To 25.
Subtract 3.5	Add 5.1
Head Diameter 25.0	30.1 Bung Diamet.

The Content, Length, Proportion of Diameters, and Variety of a Cask being given; to find the Diameters.

Find a Mean Diameter, between the Proportions of the Diameters: Thus,

Against the Length on C, set the Gauge Point on D; then against the Content on C, is the Mean Diameter on D.

Against the Mean between the Proportions on A, set the Mean Diameter of the Cask on B; then against each Proportion on A, is the corresponding Diameter on B,

Let there be a Cask of the first Variety, whose Content is 45 Wine Gallons, the Length 30, and the Proportion of Diameters as 4 to 5.

Against the Difference of Diameters, on the Line of Inches, is 0.63 on the Line of the first Variety;

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Variety; which added to 4, the lesser Proportion, the Sum is 4.63, the Mean between the Proportions.

Against 30 on C, set W. G. on D; then against 45 on C, is 21 on D, the Mean Diameter.

Against 4.63 on A, set 21 on B; then against 4 on A, is 18.1 on B, the Head Diameter; and at the same Time, against 5 on A, is 22.6 on B, the Bung Diameter.

The Content of a Cask, the Proportions of the Diameters and Depth, and the Variety being given; to find the Diameters and Depth.

Work with the Proportions, as if they were the real Dimensions of a Cask; then against the Content thus found on E, set any of the given Proportions on D, and against the Content of the Cask on E, is the corresponding Dimension on D.

Let there be a Cask of the second Variety, whose Content is 26.2 Ale Gallons, the Head, Bung and Length in the same Proportion, as 6, 7, and 8, or which is all one, as 12, 14 and 16.

Against 2, the Difference of the Proportions of the Diameters, on the Line of Inches, is 1.18 on the Line of the second Variety; which added to 12, the Proportion of the Head, the Sum is 13.18, the Mean Proportion.

Set the Gauge Point on D, against 16 on C; then against 13.18 on D, is 7.7 on C.

Against 7.7 on E, set 12 on D; then against 26.2 on E, is 18 on D, the Head Diameter.

Against 7.7 on E, set 14 on D; then against 26.2 on E, is 21 on D, the Bung Diameter.

Against 7.7 on E, set 16 on D; then against 26.2 on E, is 24 on D, the Length.

N.B. It is always most commodious, to take your Proportions in Numbers greater than 10; because

cause the Content will be found in Gallons, which will render the Operation more intelligible to the Learner, than when the Content found by the Proportions, is a Decimal.

The Content of a Cask, Head Diameter, Length and Variety being given; to find the Bung Diameter.

Set the Gauge Point on D, against the Length on C; then against the Content on C, is the Mean Diameter on D.

From the Mean Diameter, subtract the Head Diameter, and seek the Remainder on the Line of the Variety of the Cask; against which, on the Line of Inches, is the Difference of Diameters; which added to the Head Diameter, the Sum is the Bung Diameter.

Let there be a Cask the Frustum of a Spheroid, the Content of which is 87.3 Ale Gallons, the Head Diameter 25.2, and the Length 40; the Bung Diameter is required.

Against 40 on C, set A. G. on D; then against 87.3 on C, is the Mean Diameter 28 on D.

From 28.0 the Mean Diameter.
Subtract 25.2 the Head Diameter.

2.8 the Remainder.

Against 2.8 on the Spheroid Line is 4.1, the Difference of Diameters on the Line of Inches; which added to 25.2 the Head Diameter, the Sum is 29.3, the Bung Diameter.

The Content of a Cask, Bung Diameter, Length, and Variety being given; to find the Head Diameter.

Find the Mean Diameter, as in the last Proposition, and subtract it from the Bung Diameter, and note the Remainder: Then on the Line of the Variety of the Cask, seek a Number, which subtracted

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ted from the Inches and Parts against which it stands, shall leave the said Remainder; the Number thus found, subtracted from the Mean Diameter, leaves the Head Diameter.

Let the given Cask be of the first Variety, the Content 100 Wine Gallons, the Bung Diameter 30.4, and the Length 36.5; the Head Diameter is required.

Against 36.5 on C, set W. G. on D; then against 100 on C, is 28.4 on D, the Mean Diameter.

From 30.4 the Bung Diameter.
Subtract 28.4 the Mean Diameter.

2.0 the Remainder.

Now on the Line of the first Variety, find 3.6, against which is 5.6 on the Line of Inches, the Difference between which two Numbers is 2.0 the same as the Remainder; therefore subtract 3.6 from 28.4, the Mean Diameter, the Remainder is 24.8 the Head Diameter.

The Content of a Cask, the Head and Bung Diameters, and the Variety being given; to find the Length.

Set the Mean Diameter on D, against the Content on C; then against the Gauge Point on D, is the Length on C.

Let the given Cask be the Frustum of a Spheroid, the Head Diameter 25.2 the Bung Diameter 29.3, and Content 104 Wine Gallons; the Length is required.

Against 4.1, the Difference of the Diameters on the Line of Inches, is 2.8 on the Spheroid Line, which added to 25.2, the Head Diameter, the Sum is 28, the Mean Diameter.

Set

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Set 28 on D, against 104 on C; then against W. G. on D, is 39 on C, the Length.

The Bung of a lying Cask, or Length of a standing Cask, the Wet Inches, and Quantity of Liquor in the Cask being given; to find the whole Content of the Cask.

Set the Bung Diameter, or Length on Numbers, against 100 on the Segments; then against the Wet Inches on the Numbers, is a fourth Number on the Segments.

Against this fourth Number on A, set the given Quantity on B; then against 100 on A, is the whole Content on B.

Let there be a lying Cask, whose Bung Diameter is 23, the Wet Inches 11, and the Quantity in the Cask 17 Gallons; the whole Content of the Cask is required.

Set 23 on Numbers, against 100 on S. L.; then against 11 on Numbers, is 47 on S. L.

Against 47 on A, set 17 on B; then against 100 on A, is 36 on B, the whole Content.

The Bung Diameter of a lying Cask, or Length of a standing Cask, the Quantity of Liquor in the Cask and whole Content of the Cask being given; to find the Wet Inches.

Against the whole Content on A, set 100 on B; then against the Quantity in the Cask on A, is a fourth Number on B.

Against 100 on the Segments, set the Bung Diameter, or Length on the Numbers; then against the fourth Number, on the Segments, is the Wet Inches on the Numbers.

Let there be a standing Cask, whose Length is 46.5, the whole Content 110 Gallons, the Quantity of Liquor in the Cask 74 Gallons; the Wet Inches are required.

Against

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Against 110 on A, set 100 on B; then against 74 on A, is 67.5 on B.

Against 100 on S. S. set 46.5 on the Numbers; then against 67.5 on S. S. is 30.7 on Num. the Wet Inches.

The whole Content of a Cask, the Quantity of Liquor therein, and the Wet Inches being given; to find the Bung, or Length.

Against the whole Content on A, set 100 on B; then against the Quantity in the Cask on A, is a fourth Number on B.

Against the fourth Number on the Segments, set the Wet Inches on the Numbers; then against 100 on the Segments, is the Bung Diameter, or Length on the Numbers.

Let there be a Cask, whose Content is 125 Gallons, the Quantity in the Cask 48 Gallons, and Wet Inches 20; this being a standing Cask, the Length is required.

Against 125 on A, set 100 on B; then against 48 on A, is 38.5 on B.

Against 38.5 on S. S. set 20 on Num. then against 100 on S. S. is 50.3 on Num. the Length.

Of Mixing different Qualities.

The Prices of two Liquors being given or known; to find in what Proportion they must be mix'd, to sell at any given Price, between the two Prices.

The Difference between the greatest Price, and the given Price, is the Quantity of the worst Liquor. And the Difference between the given Price, and the smallest Price, is the Quantity of the best Liquor.

Let it be required to mix Brandy at 4s. 6d. the Gallon, with Brandy at 9s. the Gallon; the Proportion is required.

Greatest

Greatest Price	^{s.} 9	Given Price	^{s.} 7 $\frac{1}{2}$
Given Price	$7 \frac{1}{2}$	Smallest Price	$4 \frac{1}{2}$
Worst Liquor	$1 \frac{1}{2}$	Best Liquor	3

Thus it is found there must be put 3 Gallons of the best Brandy, to $1 \frac{1}{2}$ of the worst Brandy ; or which is the same, 2 of the best, to 1 of the worst.

The Quantity of the Mixture, the Price thereof, and the Price of each Liquor being given ; to find the Quantity of each to be mixed.

Multiply the given Quantity, by the Difference between the Prices of it, and that of the worst Quality, and divide the Product by the Difference between the best and worst Price ; the Quotient will be the Quantity of the best Liquor.

Let it be required to mix 40 Gallons of Brandy at 8s. 6d. with Brandy at 3s. 9d. to sell at the Rate of 5s. 4d. the Gallon.

	^{s. d.}		^{s. d.}
Given Price	5 4	Greatest Price	8 6
Worst Price	<u>3 9</u>	Worst Price	<u>3 9</u>
	1 7 or 19 Pence.		4 9 or 57 Pence.

$$\begin{array}{r}
 40 \text{ Given Quantity.} \\
 \underline{19} \\
 360 \\
 \underline{40} \\
 57) 760 \text{ (} 13 \frac{12}{17} \text{ Best Brandy.} \\
 \underline{57} \\
 190 \\
 \underline{171} \\
 19
 \end{array}$$

Thus it is found, that in 40 Gallons, there must be put $13 \frac{12}{17}$, or $13 \frac{1}{3}$ Gallons of the best Brandy.

X

A

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*A TABLE of EXCISE, for Common Brewers,
from one Firkin of Beer, or Ale, to ten thousand
Barrels.*

Bar.	X.				A.				VI.			
	l.	s.	d.	qrs.	l.	s.	d.	qrs.	l.	s.	d.	qrs.
$\frac{1}{4}$	0	1	1	0.17	0	1	1	2.54	0	0	3	1.91
$\frac{1}{2}$	0	2	2	0.34	0	2	3	1.09	0	0	6	3.82
$\frac{3}{4}$	0	3	3	0.51	0	3	4	3.63	0	0	10	1.73
1	0	4	4	0.69	0	4	6	2.18	0	1	1	3.65
2	0	8	8	1.39	0	9	1	0.36	0	2	3	3.30
3	0	13	0	2.08	0	13	7	2.54	0	3	5	2.95
4	0	17	4	2.78	0	18	2	0.72	0	4	7	2.60
5	1	1	8	3.47	1	2	8	2.90	0	5	9	2.26
6	1	6	1	0.17	1	7	3	1.09	0	6	11	1.91
7	1	10	5	0.86	1	11	9	3.27	0	8	1	1.56
8	1	14	9	1.56	1	16	4	1.45	0	9	3	1.21
9	1	19	1	2.26	2	0	10	3.63	0	10	5	0.86
10	2	3	5	2.95	2	5	5	1.81	0	11	7	0.52
11	2	7	9	3.65	2	10	0	0.00	0	12	9	0.17
12	2	12	2	0.34	2	14	6	2.18	0	13	10	3.82
13	2	16	6	1.04	2	19	1	0.36	0	15	0	3.47
14	3	0	10	1.73	3	3	7	2.54	0	16	2	3.13
15	3	5	2	2.43	3	8	2	0.72	0	17	4	2.78
16	3	9	6	3.13	3	12	8	2.90	0	18	6	2.43
17	3	13	10	3.82	3	17	3	1.09	0	19	8	2.08
18	3	18	3	0.52	4	1	9	3.27	1	0	10	1.73
19	4	2	7	1.21	4	6	4	1.45	1	2	0	1.39
20	4	6	11	1.91	4	10	10	3.63	1	3	2	1.04
21	4	11	3	2.60	4	15	5	1.81	1	4	4	0.69
22	4	15	7	3.30	5	0	0	0.00	1	5	6	0.34
23	5	0	0	0.00	5	4	6	2.18	1	6	8	0.00
24	5	4	4	0.69	5	9	1	0.36	1	7	9	3.65
25	5	8	8	1.39	5	13	7	2.54	1	8	11	3.30
26	5	13	0	2.08	5	18	2	0.72	1	10	1	2.95
27	5	17	4	2.78	6	2	8	2.90	1	11	3	2.60
28	6	1	8	3.47	6	7	3	1.09	1	12	5	2.26
29	6	6	1	0.17	6	11	9	3.27	1	13	7	1.91
30	6	10	5	0.86	6	16	4	1.45	1	14	9	1.56
31	6	14	9	1.56	7	0	10	3.63	1	15	11	1.21
32	6	19	1	2.26	7	5	5	1.81	1	17	1	0.86
33	7	3	5	2.95	7	10	0	0.00	1	18	3	0.52
34	7	7	9	3.65	7	14	6	2.18	1	19	5	0.17
35	7	12	2	0.34	7	19	1	0.36	2	0	6	3.82
36	7	16	6	1.04	8	3	7	2.54	2	1	8	3.47
37	8	0	10	1.73	8	8	2	0.72	2	2	10	3.13
38	8	5	2	2.43	8	12	8	2.90	2	4	0	2.78
39	8	9	6	3.13	8	17	3	1.09	2	5	2	2.43
40	8	13	10	3.82	9	1	9	3.27	2	6	4	2.08

A TABLE of EXCISE for Common Brewers,
from one Firkin of Beer, or Ale, to ten thousand
Barrels.

Bar.	X.				A.				VI.			
	l.	s.	d.	qrs.	l.	s.	d.	qrs.	l.	s.	d.	qrs.
41	8	18	3	0.52	9	6	4	1.45	2	7	6	1.73
42	9	2	7	1.21	9	10	10	3.63	2	8	8	1.39
43	9	6	11	1.91	9	15	5	1.81	2	9	10	1.94
44	9	11	3	2.60	10	0	0	0.00	2	11	0	0.69
45	9	15	7	3.30	10	4	6	2.18	2	12	2	0.34
46	10	0	0	0.00	10	9	1	0.36	2	13	4	0.90
47	10	4	4	0.69	10	13	7	2.54	2	14	5	3.65
48	10	8	8	1.39	10	18	2	0.72	2	15	7	3.30
49	10	13	0	2.08	11	2	8	2.90	2	16	9	2.95
50	10	17	4	2.78	11	7	3	1.09	2	17	11	2.60
51	11	1	8	3.47	11	11	9	3.27	2	19	1	2.26
52	11	6	1	0.17	11	16	4	1.45	3	0	3	1.91
53	11	10	5	0.86	12	0	10	3.63	3	1	5	1.56
54	11	14	9	1.56	12	5	5	1.81	3	2	7	1.21
55	11	19	1	2.26	12	10	0	0.00	3	3	9	0.86
56	12	3	5	2.95	12	14	6	2.18	3	4	11	0.52
57	12	7	9	3.65	12	19	1	0.36	3	6	1	0.17
58	12	12	2	0.34	13	3	7	2.54	3	7	2	3.82
59	12	16	6	1.04	13	8	2	0.72	3	8	4	3.47
60	13	0	10	1.73	13	12	8	2.90	3	9	6	3.13
61	13	5	2	2.43	13	17	3	1.09	3	10	8	2.78
62	13	9	6	3.13	14	1	9	3.27	3	11	10	2.43
63	13	13	10	3.82	14	6	4	1.45	3	13	0	2.08
64	13	18	3	0.52	14	10	10	3.63	3	14	2	1.73
65	14	2	7	1.21	14	15	5	1.81	3	15	4	1.39
66	14	6	11	1.91	15	0	0	0.00	3	16	6	1.04
67	14	11	3	2.60	15	4	6	2.18	3	17	8	0.69
68	14	15	7	3.30	15	9	1	0.36	3	18	10	0.34
69	15	0	0	0.00	15	13	7	2.54	4	0	0	0.00
70	15	4	4	0.69	15	18	2	0.72	4	1	1	3.65
71	15	8	8	1.39	16	2	8	2.90	4	2	3	3.30
72	15	13	0	2.08	16	7	3	1.09	4	3	5	2.95
73	15	17	4	2.78	16	11	9	3.27	4	4	7	2.60
74	16	1	8	3.47	16	16	4	1.45	4	5	9	2.26
75	16	6	1	0.17	17	0	10	3.63	4	6	11	1.91
76	16	10	5	0.86	17	5	5	1.81	4	8	1	1.56
77	16	14	9	1.56	17	10	0	0.00	4	9	3	1.21
78	16	19	1	2.26	17	14	6	2.18	4	10	5	0.86
79	17	3	5	2.95	17	19	1	0.36	4	11	7	0.52
80	17	7	9	3.65	18	3	7	2.54	4	12	9	0.17
81	17	12	2	0.34	18	8	2	0.72	4	13	10	3.82
82	17	16	6	1.04	18	12	8	2.90	4	15	0	3.47
83	18	0	10	1.73	18	17	3	1.09	4	16	2	3.13
84	18	5	2	2.43	19	1	9	3.27	4	17	4	2.78
85	18	9	6	3.13	19	6	4	1.45	4	18	6	2.43

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A TABLE of EXCISE for Common Brewers, from one Firkin of Beer, or Ale, to ten thousand Barrels.

Bar.	X.				A.				VI.			
	l.	s.	d.	qrs.	l.	s.	d.	qrs.	l.	s.	d.	qrs.
86	18	13	10	3.82	19	10	10	3.63	4	19	8	2.08
87	18	16	3	0.52	19	15	5	1.81	5	0	10	1.73
88	19	2	7	1.21	20	0	0	0.00	5	2	0	1.39
89	19	6	11	1.91	20	4	6	2.18	5	3	2	1.04
90	19	11	3	2.60	20	9	1	0.36	5	4	4	0.69
91	19	15	7	3.30	20	13	7	2.54	5	5	6	0.34
92	20	0	0	0.00	20	18	2	0.72	5	6	8	0.00
93	20	4	4	0.69	21	2	8	2.90	5	7	9	3.65
94	20	8	8	1.39	21	7	3	1.09	5	8	11	3.30
95	20	13	0	2.08	21	11	9	3.27	5	10	1	2.95
96	20	17	4	2.78	21	16	4	1.45	5	11	3	2.60
97	21	1	8	3.47	22	0	10	3.63	5	12	5	2.26
98	21	6	1	0.17	22	5	5	1.81	5	13	7	1.91
99	21	10	5	0.86	22	10	0	0.00	5	14	9	1.56
100	21	14	9	1.56	22	14	9	2.18	5	15	11	1.21
200	43	9	6	3.13	45	9	1	0.36	11	11	10	2.43
300	65	4	4	0.69	68	3	7	2.54	17	7	9	3.65
400	86	19	1	2.26	90	18	2	0.72	23	3	9	0.86
500	108	13	10	3.82	113	12	8	2.90	28	19	8	2.08
600	130	8	8	1.39	136	7	3	1.09	34	15	7	3.30
700	152	3	5	2.95	159	1	9	3.27	40	11	7	0.52
800	173	18	3	0.52	181	16	4	1.45	46	7	6	1.73
900	195	13	0	2.08	204	10	10	3.63	52	3	5	2.95
1000	217	7	9	3.65	227	5	5	1.81	57	19	5	0.17
2000	434	15	7	3.30	454	10	10	3.63	115	18	10	0.34
3000	652	3	5	2.95	681	16	4	1.45	173	18	3	0.52
4000	869	11	3	2.60	909	1	9	3.27	231	17	8	0.69
5000	1086	19	1	2.26	1136	7	3	1.09	289	17	1	0.86
6000	1304	6	11	1.91	1363	12	8	2.90	347	16	6	1.04
7000	1521	14	9	1.56	1590	18	2	0.72	405	15	11	1.21
8000	1739	2	7	1.21	1818	3	7	2.54	463	15	4	1.39
9000	1956	10	5	0.87	2045	9	1	0.36	521	14	9	1.56
10000	2173	18	3	0.52	2272	14	6	2.18	579	14	2	1.73

The

The Use of the foregoing Table of Excise for Common Brewers.

The two Figures on the Right-Hand of the Place of Farthings, are Decimal Fractions of a Farthing; therefore seek the Number of Barrels, and Firkins, and take out their corresponding Sums, in Pounds, Shillings, Pence, Farthings and Decimal Parts of a Farthing: Add these Sums together, the Total is the Excise.

What's the Excise of sixty-eight Barrels, and one Firkin of Strong Beer?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
Against 68 Barrels is	14	15	7	3.30
Against 1 Firkin is	0	1	1	0.17
	14	16	8	3.47

Thus it is found, that the Excise of 68 Barrels, and 1 Firkin of Strong Beer, is 14*l.* 16*s.* 8*d.* $\frac{3}{4}$ and $\frac{47}{100}$ of a Farthing.

What's the Duty of three hundred twenty-four Barrels, and three Firkins of Ale?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
Against 300 Barrels is	68	3	7	2.54
Against 24 Barrels is	5	9	1	0.36
Against 3 Firkins is	0	3	4	3.63
	73	16	1	2.53

The Duty is 73*l.* 16*s.* 1*d.* $\frac{1}{2}$, and $\frac{53}{100}$ of a Farthing.

What's the Duty of two thousand and twenty-eight Barrels of Small Beer?

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>qrs.</i>
Against 2000 Barrels is	115	18	10	0.34
Against 28 Barrels is	1	12	5	2.26
	117	11	3	2.60

The Duty is 117*l.* 11*s.* 3*d.* $\frac{1}{2}$, and $\frac{60}{100}$ of a Farthing.

A TABLE of EXCISE for ViEtuallers, from one Firkin of Strong and Small Beer to 10,000 Barrels.

Bar.	X.			VI.			Bar.	X.			VI.		
	l.	s.	d.	l.	s.	d.		l.	s.	d.	l.	s.	d.
$\frac{1}{4}$	0	1	3	0	0	4	31	7	15	0	2	1	4
$\frac{1}{2}$	0	2	6	0	0	8	32	8	0	0	2	2	8
$\frac{3}{4}$	0	3	9	0	1	0	33	8	5	0	2	4	0
1	0	5	0	0	1	4	34	8	10	0	2	5	4
2	0	10	0	0	2	8	35	8	15	0	2	6	8
3	0	15	0	0	4	0	36	9	0	0	2	8	0
4	1	0	0	0	5	4	37	9	5	0	2	9	4
5	1	5	0	0	6	8	38	9	10	0	2	10	8
6	1	10	0	0	8	0	39	9	15	0	2	12	0
7	1	15	0	0	9	4	40	10	0	0	2	13	4
8	2	0	0	0	10	8	41	10	5	0	2	14	8
9	2	5	0	0	12	0	42	10	10	0	2	16	0
10	2	10	0	0	13	4	43	10	15	0	2	17	4
11	2	15	0	0	14	8	44	11	0	0	2	18	8
12	3	0	0	0	16	0	45	11	5	0	3	0	0
13	3	5	0	0	17	4	46	11	10	0	3	1	4
14	3	10	0	0	18	8	47	11	15	0	3	2	8
15	3	15	0	1	0	0	48	12	0	0	3	4	0
16	4	0	0	1	1	4	49	12	5	0	3	5	4
17	4	5	0	1	2	8	50	12	10	0	3	6	8
18	4	10	0	1	4	0	51	12	15	0	3	8	0
19	4	15	0	1	5	4	52	13	0	0	3	9	4
20	5	0	0	1	6	8	53	13	5	0	3	10	8
21	5	5	0	1	8	0	54	13	10	0	3	12	0
22	5	10	0	1	9	4	55	13	15	0	3	13	4
23	5	15	0	1	10	8	56	14	0	0	3	14	8
24	6	0	0	1	12	0	57	14	5	0	3	16	0
25	6	5	0	1	13	4	58	14	10	0	3	17	4
26	6	10	0	1	14	8	59	14	15	0	3	18	8
27	6	15	0	1	16	0	60	15	0	0	4	0	0
28	7	0	0	1	17	4	61	15	5	0	4	1	4
29	7	5	0	1	18	8	62	15	10	0	4	2	8
30	7	10	0	2	0	0	63	15	15	0	4	4	0
							64	16	0	0	4	5	4
							65	16	5	0	4	6	8

A TABLE of EXCISE for Vituallers, from one Firkin of Strong and Small Beer to 10,000 Barrels.

X.				VI.			
Bar.	l.	s.	d.	l.	s.	d.	
66	16	10	0	4	8	0	
67	16	15	0	4	9	4	
68	17	0	0	4	10	8	
69	17	5	0	4	12	0	
70	17	10	0	4	13	4	
71	17	15	0	4	14	8	
72	18	0	0	4	16	0	
73	18	5	0	4	17	4	
74	18	10	0	4	18	8	
75	18	15	0	5	0	0	
76	19	0	0	5	1	4	
77	19	5	0	5	2	8	
78	19	10	0	5	4	0	
79	19	15	0	5	5	4	
80	20	0	0	5	6	8	
81	20	5	0	5	8	0	
82	20	10	0	5	9	4	
83	20	15	0	5	10	8	
84	21	0	0	5	12	0	
85	21	5	0	5	13	4	
86	21	10	0	5	14	8	
87	21	15	0	5	16	0	
88	22	0	0	5	17	4	
89	22	5	0	5	18	8	
90	22	10	0	6	0	0	
91	22	15	0	6	1	4	
92	23	0	0	6	2	8	
93	23	5	0	6	4	0	
94	23	10	0	6	5	4	
95	23	15	0	6	6	8	

X.				VI.			
Bar.	l.	s.	d.	l.	s.	d.	
96	24	0	0	6	8	0	
97	24	5	0	6	9	4	
98	24	10	0	6	10	8	
99	24	15	0	6	12	0	
100	25	0	0	6	13	4	
200	50	0	0	13	6	8	
300	75	0	0	20	0	0	
400	100	0	0	26	13	4	
500	125	0	0	33	6	8	
600	150	0	0	40	0	0	
700	175	0	0	46	13	4	
800	200	0	0	53	6	8	
900	225	0	0	60	0	0	
1000	250	0	0	66	13	4	
2000	500	0	0	133	6	8	
3000	750	0	0	200	0	0	
4000	1000	0	0	266	13	4	
5000	1250	0	0	333	6	8	
6000	1500	0	0	400	0	0	
7000	1750	0	0	466	13	4	
8000	2000	0	0	533	6	8	
9000	2250	0	0	600	0	0	
10000	2500	0	0	666	13	4	



*The Use of the foregoing Table of Excise for
Viſtuallers.*

What's the Duty of 78 Barrels of Strong Beer?

Against 78 Barrels is 19 l. 10 s. the Duty.

*What's the Duty of 48 Barrels, and three Firkins
of Strong Beer?*

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Against 48 Barrels is	12	0	0
Against 3 Firkins is	0	3	9

12 3 9 the Duty.

*What's the Duty of 214 Barrels, and 2 Firkins
of Small Beer?*

	<i>l.</i>	<i>s.</i>	<i>d.</i>
Against 200 Barrels is	13	6	8
Against 14 Barrels is	0	18	8
Against 2 Firkins is	0	0	8

14 6 0 the Duty.

A
T A B L E
O F T H E
A R E A S o f C I R C L E S,
A N D
C O N T E N T S o f C Y L I N D E R S
I N
A L E G A L L O N S,

To all Diameters in Inches and Tenths, from
twelve Inches, to One hundred and fifty-six.

Y

Areas

162 *The Young GAUGER'S Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
12 .0	0.4011	0.80	1.20	1.60
.1	0.4078	0.81	1.22	1.63
.2	0.4146	0.83	1.24	1.65
.3	0.4214	0.84	1.26	1.69
.4	0.4282	0.86	1.28	1.71
.5	0.4352	0.87	1.30	1.74
.6	0.4422	0.88	1.33	1.77
.7	0.4492	0.90	1.35	1.80
.8	0.4563	0.91	1.37	1.82
.9	0.4635	0.93	1.39	1.85
13 .0	0.4707	0.94	1.41	1.88
.1	0.4780	0.96	1.43	1.91
.2	0.4853	0.97	1.46	1.94
.3	0.4927	0.98	1.48	1.97
.4	0.5001	1.00	1.50	2.00
.5	0.5076	1.01	1.52	2.03
.6	0.5151	1.03	1.55	2.06
.7	0.5227	1.04	1.57	2.09
.8	0.5304	1.06	1.59	2.12
.9	0.5381	1.08	1.61	2.15
14 .0	0.5459	1.09	1.64	2.18
.1	0.5537	1.11	1.66	2.21
.2	0.5616	1.12	1.68	2.24
.3	0.5695	1.14	1.71	2.28
.4	0.5775	1.15	1.73	2.31
.5	0.5856	1.17	1.76	2.34
.6	0.5937	1.19	1.78	2.37
.7	0.6018	1.20	1.80	2.40
.8	0.6100	1.22	1.83	2.44
.9	0.6183	1.24	1.85	2.47
15 .0	0.6266	1.25	1.88	2.50
.1	0.6350	1.27	1.91	2.54
.2	0.6435	1.29	1.93	2.57
.3	0.6520	1.30	1.96	2.61
.4	0.6605	1.32	1.98	2.64
.5	0.6691	1.34	2.01	2.68
.6	0.6778	1.35	2.03	2.71
.7	0.6865	1.37	2.06	2.74
.8	0.6953	1.39	2.09	2.78
.9	0.7041	1.41	2.11	2.82

Areas

The Young GAUGER's Instructor. 163

in ALE GALLONS.				
Depth.				
5	6	7	8	9
2.01	2.41	2.81	3.21	3.61
2.04	2.44	2.85	3.26	3.67
2.07	2.49	2.90	3.31	3.73
2.11	2.53	2.95	3.37	3.79
2.14	2.57	3.00	3.43	3.86
2.18	2.61	3.05	3.48	3.92
2.21	2.65	3.10	3.54	3.98
2.24	2.69	3.14	3.59	4.04
2.28	2.73	3.19	3.65	4.10
2.32	2.78	3.24	3.71	4.17
2.35	2.82	3.30	3.77	4.24
2.39	2.87	3.35	3.82	4.30
2.43	2.91	3.39	3.88	4.36
2.46	2.95	3.44	3.94	4.43
2.50	3.00	3.50	4.00	4.50
2.54	3.04	3.55	4.06	4.56
2.58	3.09	3.61	4.12	4.63
2.61	3.13	3.66	4.18	4.70
2.65	3.18	3.71	4.24	4.77
2.69	3.23	3.77	4.30	4.84
2.73	3.27	3.82	4.36	4.91
2.77	3.32	3.87	4.42	4.98
2.81	3.37	3.93	4.49	5.05
2.84	3.41	3.98	4.55	5.12
2.88	3.46	4.04	4.62	5.19
2.93	3.51	4.10	4.68	5.26
2.97	3.56	4.15	4.74	5.34
3.01	3.61	4.21	4.81	5.41
3.05	3.66	4.27	4.88	5.49
3.09	3.71	4.33	4.95	5.56
3.13	3.76	4.39	5.01	5.64
3.18	3.81	4.45	5.08	5.71
3.22	3.86	4.50	5.15	5.79
3.26	3.91	4.56	5.22	5.87
3.30	3.96	4.62	5.28	5.94
3.35	4.01	4.68	5.35	6.02
3.39	4.06	4.74	5.42	6.10
3.43	4.12	4.80	5.49	6.18
3.48	4.17	4.87	5.56	6.26
3.52	4.22	4.93	5.63	6.34

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
16.0	0.7130	1.43	2.14	2.85
.1	0.7219	1.44	2.16	2.89
.2	0.7309	1.46	2.19	2.92
.3	0.7400	1.48	2.22	2.96
.4	0.7491	1.50	2.25	3.00
.5	0.7582	1.52	2.27	3.03
.6	0.7674	1.53	2.30	3.07
.7	0.7767	1.55	2.33	3.10
.8	0.7861	1.57	2.36	3.14
.9	0.7954	1.59	2.39	3.18
17.0	0.8049	1.61	2.41	3.22
.1	0.8144	1.63	2.44	3.26
.2	0.8239	1.65	2.47	3.29
.3	0.8335	1.67	2.50	3.33
.4	0.8432	1.69	2.53	3.37
.5	0.8529	1.70	2.56	3.41
.6	0.8627	1.72	2.59	3.45
.7	0.8725	1.74	2.62	3.49
.8	0.8824	1.76	2.65	3.53
.9	0.8924	1.78	2.68	3.57
18.0	0.9024	1.80	2.71	3.61
.1	0.9124	1.82	2.74	3.65
.2	0.9225	1.84	2.77	3.69
.3	0.9327	1.87	2.80	3.73
.4	0.9429	1.89	2.83	3.77
.5	0.9532	1.91	2.86	3.81
.6	0.9635	1.93	2.89	3.85
.7	0.9739	1.95	2.92	3.90
.8	0.9843	1.97	2.95	3.94
.9	0.9948	1.99	2.98	3.98
19.0	1.0054	2.01	3.02	4.02
.1	1.0160	2.03	3.05	4.06
.2	1.0267	2.05	3.08	4.11
.3	1.0374	2.07	3.11	4.15
.4	1.0482	3.10	3.14	4.19
.5	1.0590	2.12	3.18	4.24
.6	1.0699	2.14	3.21	4.28
.7	1.0808	2.16	3.24	4.32
.8	1.0918	2.18	3.27	4.37
.9	1.1029	2.21	3.31	4.41

The Young GAUGER'S Instructor. 165

in ALE GALLONS.				
Depth.				
5	6	7	8	9
3.57	4.28	4.99	5.70	6.42
3.61	4.33	5.05	5.78	6.50
3.65	4.39	5.12	5.85	6.58
3.70	4.44	5.18	5.92	6.66
3.75	4.49	5.24	5.99	6.74
3.79	4.55	5.31	6.06	6.82
3.84	4.60	5.37	6.14	6.90
3.88	4.66	5.43	6.21	6.99
3.93	4.71	5.50	6.28	7.07
3.98	4.77	5.57	6.36	7.16
4.02	4.83	5.63	6.44	7.24
4.07	4.88	5.70	6.51	7.33
4.12	4.94	5.77	6.59	7.42
4.17	5.00	5.83	6.66	7.50
4.22	5.06	5.90	6.74	7.59
4.26	5.11	5.97	6.82	7.68
4.31	5.17	6.03	6.90	7.76
4.36	5.23	6.10	6.98	7.85
4.41	5.29	6.27	7.06	7.94
4.46	5.35	6.24	7.14	8.03
4.51	5.41	6.31	7.22	8.12
4.56	5.47	6.38	7.30	8.21
4.61	5.53	6.45	7.38	8.30
4.66	5.59	6.53	7.46	8.39
4.71	5.66	6.60	7.54	8.49
4.77	5.72	6.67	7.63	8.58
4.82	5.78	6.74	7.71	8.67
4.87	5.84	6.82	7.79	8.77
4.92	5.90	6.89	7.87	8.86
4.97	5.97	6.96	7.95	8.95
5.03	6.03	7.04	8.04	9.05
5.08	6.10	7.11	8.13	9.14
5.13	6.16	7.18	8.21	9.23
5.19	6.22	7.26	8.30	9.34
5.24	6.29	7.34	8.38	9.43
5.30	6.35	7.41	8.47	9.53
5.35	6.42	7.49	8.56	9.63
5.40	6.49	7.57	8.65	9.73
5.46	6.55	7.65	8.74	9.83
5.51	6.62	7.72	8.82	9.93

Areas

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
20 .0	1.1140	2.23	3.34	4.46
.1	1.1252	2.25	3.38	4.50
.2	1.1364	2.27	3.41	4.55
.3	1.1477	2.29	3.44	4.59
.4	1.1590	2.32	3.48	4.64
.5	1.1704	2.34	3.51	4.68
.6	1.1819	2.36	3.54	4.73
.7	1.1934	2.39	3.58	4.77
.8	1.2049	2.41	3.61	4.82
.9	1.2165	2.43	3.65	4.86
21 .0	1.2282	2.46	3.68	4.91
.1	1.2400	2.48	3.72	4.96
.2	1.2517	2.50	3.75	5.01
.3	1.2635	2.53	3.79	5.05
.4	1.2754	2.55	3.83	5.10
.5	1.2874	2.57	3.86	5.15
.6	1.2994	2.60	3.90	5.20
.7	1.3114	2.62	3.93	5.25
.8	1.3236	2.65	3.97	5.29
.9	1.3357	2.67	4.01	5.34
22 .0	1.3480	2.70	4.04	5.39
.1	1.3602	2.72	4.08	5.44
.2	1.3726	2.75	4.12	5.49
.3	1.3850	2.77	4.16	5.54
.4	1.3974	2.79	4.19	5.59
.5	1.4099	2.82	4.23	5.64
.6	1.4225	2.84	4.27	5.69
.7	1.4351	2.87	4.31	5.74
.8	1.4478	2.90	4.34	5.79
.9	1.4605	2.92	4.38	5.84
23 .0	1.4733	2.95	4.42	5.89
.1	1.4861	2.97	4.46	5.94
.2	1.4990	3.00	4.50	6.00
.3	1.5120	3.02	4.54	6.05
.4	1.5250	3.05	4.58	6.10
.5	1.5380	3.08	4.61	6.15
.6	1.5511	3.10	4.65	6.20
.7	1.5643	3.13	4.69	6.26
.8	1.5775	3.15	4.73	6.31
.9	1.5908	3.18	4.77	6.36

in ALE GALLONS.				
Depth.				
5	6	7	8	9
5.57	6.68	7.80	8.91	10.03
5.63	6.75	7.88	9.00	10.13
5.68	6.82	7.95	9.09	10.23
5.74	6.88	8.03	9.18	10.33
5.80	6.95	8.11	9.27	10.43
5.85	7.02	8.19	9.36	10.53
5.91	7.09	8.27	9.46	10.64
5.97	7.16	8.35	9.55	10.74
6.02	7.23	8.43	9.64	10.84
6.08	7.30	8.51	9.73	10.94
6.14	7.37	8.60	9.82	11.05
6.20	7.44	8.68	9.92	11.16
6.26	7.51	8.76	10.01	11.26
6.32	7.58	8.84	10.11	11.37
6.38	7.65	8.93	10.20	11.48
6.44	7.72	9.01	10.30	11.59
6.50	7.80	9.09	10.39	11.69
6.56	7.87	9.18	10.49	11.80
6.62	7.94	9.26	10.59	11.91
6.68	8.01	9.35	10.69	12.02
6.74	8.09	9.44	10.78	12.13
6.80	8.16	9.52	10.88	12.24
6.86	8.23	9.61	10.98	12.35
6.93	8.31	9.70	11.08	12.47
6.99	8.38	9.78	11.18	12.58
7.05	8.46	9.87	11.28	12.69
7.11	8.53	9.96	11.38	12.80
7.18	8.61	10.05	11.48	12.91
7.24	8.68	10.13	11.58	13.03
7.30	8.76	10.22	11.68	13.14
7.37	8.84	10.31	11.78	13.26
7.43	8.92	10.40	11.89	13.37
7.50	8.99	10.49	11.99	13.49
7.56	9.07	10.58	12.10	13.61
7.63	9.15	10.68	12.20	13.73
7.69	9.23	10.77	12.30	13.84
7.76	9.31	10.86	12.41	13.96
7.82	9.39	10.95	12.51	14.08
7.89	9.46	11.04	12.62	14.20
7.95	9.54	11.13	12.72	14.32

168 *The Young GAUGER'S Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
24 .0	1.6042	3.21	4.81	6.42
.1	1.6176	3.23	4.85	6.47
.2	1.6310	3.26	4.89	6.52
.3	1.6445	3.29	4.93	6.58
.4	1.6581	3.32	4.97	6.63
.5	1.6717	3.34	5.01	6.69
.6	1.6854	3.37	5.06	6.74
.7	1.6991	3.40	5.10	6.80
.8	1.7129	3.43	5.14	6.85
.9	1.7267	3.45	5.18	6.91
25 .0	1.7406	3.48	5.22	6.96
.1	1.7546	3.51	5.26	7.02
.2	1.7686	3.54	5.31	7.07
.3	1.7827	3.57	5.35	7.13
.4	1.7968	3.59	5.39	7.19
.5	1.8110	3.62	5.43	7.24
.6	1.8252	3.65	5.48	7.30
.7	1.8395	3.68	5.52	7.36
.8	1.8538	3.71	5.56	7.42
.9	1.8682	3.74	5.60	7.47
26 .0	1.8827	3.77	5.65	7.53
.1	1.8972	3.79	5.69	7.59
.2	1.9117	3.82	5.73	7.65
.3	1.9264	3.85	5.78	7.71
.4	1.9411	3.88	5.82	7.76
.5	1.9558	3.91	5.87	7.82
.6	1.9706	3.94	5.91	7.88
.7	1.9854	3.97	5.96	7.94
.8	2.0003	4.00	6.00	8.00
.9	2.0153	4.03	6.05	8.06
27 .0	2.0303	4.06	6.09	8.12
.1	2.0454	4.09	6.14	8.18
.2	2.0605	4.12	6.18	8.24
.3	2.0757	4.15	6.23	8.30
.4	2.0909	4.18	6.27	8.36
.5	2.1062	4.21	6.32	8.42
.6	2.1215	4.24	6.36	8.48
.7	2.1369	4.27	6.41	8.55
.8	2.1524	4.30	6.46	8.61
.9	2.1679	4.33	6.50	8.67

The Young GAUGER'S Instructor. 169

in ALE GALLONS.				
Depth.				
5	6	7	8	9
8.02	9.62	11.23	12.83	14.44
8.09	9.70	11.32	12.93	14.56
8.16	9.79	11.42	13.05	14.68
8.22	9.86	11.51	13.15	14.80
8.29	9.95	11.61	13.26	14.92
—	—	—	—	—
8.36	10.03	11.70	13.37	15.04
8.43	10.11	11.80	13.48	15.17
8.50	10.19	11.89	13.59	15.29
8.56	10.28	11.99	13.70	15.42
8.63	10.36	12.08	13.81	15.54
—	—	—	—	—
8.70	10.44	12.18	13.92	15.67
8.77	10.53	12.28	14.03	15.79
8.84	10.61	12.38	14.15	15.91
8.91	10.70	12.47	14.26	16.04
8.98	10.78	12.57	14.37	16.17
—	—	—	—	—
9.06	10.87	12.68	14.49	16.30
9.13	10.95	12.78	14.60	16.43
9.20	11.04	12.88	14.71	16.55
9.27	11.12	12.98	14.83	16.69
9.34	11.21	13.08	14.94	16.81
—	—	—	—	—
9.41	11.29	13.18	15.06	16.94
9.49	11.38	13.28	15.18	17.07
9.56	11.47	13.38	15.29	17.20
9.63	11.56	13.48	15.41	17.34
9.70	11.65	13.59	15.53	17.47
—	—	—	—	—
9.78	11.73	13.69	15.65	17.60
9.85	11.82	13.79	15.76	17.74
9.93	11.91	13.90	15.88	17.87
10.00	12.00	14.00	16.00	18.00
10.08	12.09	14.11	16.12	18.14
—	—	—	—	—
10.15	12.18	14.21	16.24	18.27
10.23	12.27	14.32	16.36	18.41
10.30	12.36	14.42	16.48	18.54
10.38	12.45	14.53	16.61	18.68
10.45	12.55	14.64	16.73	18.82
—	—	—	—	—
10.53	12.64	14.74	16.85	18.95
10.61	12.73	14.85	16.97	19.09
10.68	12.82	14.96	17.10	19.23
10.76	12.91	15.06	17.22	19.37
10.84	13.01	15.18	17.34	19.51

Z

Areas

170 *The Young GAUGER's Instructor.*

Diam. in Inch.	Areas of Circles and Contents of Cylinders			
	Depth.			
	1	2	3	4
28.0	2.1835	4.37	6.55	8.73
.1	2.1991	4.40	6.60	8.80
.2	2.2148	4.43	6.64	8.86
.3	2.2305	4.46	6.69	8.92
.4	2.2463	4.49	6.74	8.98
.5	2.2621	4.52	6.79	9.05
.6	2.2780	4.56	6.83	9.11
.7	2.2940	4.59	6.88	9.18
.8	2.3100	4.62	6.93	9.24
.9	2.3261	4.65	6.98	9.30
29.0	2.3422	4.68	7.03	9.37
.1	2.3584	4.72	7.07	9.43
.2	2.3746	4.75	7.12	9.50
.3	2.3910	4.78	7.17	9.56
.4	2.4073	4.81	7.22	9.63
.5	2.4237	4.85	7.27	9.69
.6	2.4401	4.88	7.32	9.76
.7	2.4567	4.91	7.37	9.83
.8	2.4732	4.94	7.42	9.89
.9	2.4899	4.98	7.47	9.96
30.0	2.5066	5.01	7.52	10.03
.1	2.5233	5.05	7.57	10.09
.2	2.5401	5.08	7.62	10.16
.3	2.5570	5.11	7.67	10.23
.4	2.5739	5.15	7.72	10.29
.5	2.5908	5.18	7.77	10.36
.6	2.6078	5.21	7.82	10.43
.7	2.6249	5.25	7.87	10.50
.8	2.6420	5.28	7.93	10.57
.9	2.6592	5.32	7.98	10.64
31.0	2.6764	5.35	8.03	10.71
.1	2.6938	5.39	8.08	10.78
.2	2.7111	5.42	8.13	10.84
.3	2.7285	5.46	8.19	10.91
.4	2.7460	5.49	8.24	10.98
.5	2.7635	5.53	8.29	11.05
.6	2.7811	5.56	8.34	11.12
.7	2.7987	5.60	8.40	11.19
.8	2.8163	5.63	8.45	11.27
.9	2.8341	5.67	8.50	11.34

in ALE GALLONS.				
Depth..				
5	6	7	8	9
10.92	13.10	15.28	17.46	19.65
11.00	13.19	15.39	17.59	19.79
11.08	13.28	15.50	17.71	19.93
11.15	13.38	15.61	17.84	20.07
11.23	13.48	15.72	17.97	20.22
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11.31	13.57	15.83	18.10	20.36
11.39	13.67	15.95	18.22	20.50
11.47	13.76	16.06	18.35	20.65
11.55	13.86	16.17	18.48	20.79
11.63	13.96	16.28	18.61	20.93
---	---	---	---	---
11.71	14.05	16.39	18.74	21.08
11.79	14.15	16.51	18.86	21.22
11.87	14.25	16.62	18.99	21.37
11.96	14.35	16.74	19.13	21.52
12.04	14.44	16.85	19.26	21.66
---	---	---	---	---
12.12	14.54	16.96	19.39	21.81
12.20	14.64	17.08	19.52	21.96
12.28	14.74	17.19	19.65	22.10
12.37	14.84	17.31	19.78	22.26
12.45	14.94	17.43	19.92	22.41
---	---	---	---	---
12.53	15.04	17.54	20.05	22.56
12.62	15.14	17.66	20.18	22.71
12.70	15.24	17.78	20.32	22.86
12.79	15.34	17.90	20.46	23.01
12.87	15.44	18.02	20.59	23.17
---	---	---	---	---
12.95	15.55	18.14	20.73	23.32
13.04	15.65	18.25	20.86	23.47
13.12	15.75	18.37	21.00	23.62
13.21	15.85	18.49	21.14	23.78
13.30	15.95	18.61	21.27	23.93
---	---	---	---	---
13.38	16.06	18.73	21.41	24.09
13.47	16.16	18.86	21.55	24.25
13.56	16.27	18.98	21.69	24.40
13.64	16.37	19.10	21.83	24.56
13.73	16.48	19.22	21.97	24.71
---	---	---	---	---
13.82	16.58	19.34	22.10	24.87
13.91	16.69	19.47	22.25	25.03
13.99	16.79	19.59	22.38	25.19
14.08	16.90	19.71	22.53	25.35
14.17	17.00	19.84	22.67	25.51

172 *The Young GAUGER'S Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
32 .0	2.8519	5.70	8.56	11.41
.1	2.8698	5.74	8.61	11.48
.2	2.8877	5.78	8.66	11.55
.3	2.9057	5.81	8.72	11.62
.4	2.9237	5.85	8.77	11.69
—	—	—	—	—
.5	2.9418	5.88	8.83	11.77
.6	2.9599	5.92	8.88	11.84
.7	2.9781	5.96	8.93	11.91
.8	2.9963	5.99	8.99	11.98
.9	3.0146	6.03	9.04	12.06
—	—	—	—	—
33 .0	3.0330	6.07	9.10	12.13
.1	3.0514	6.10	9.15	12.21
.2	3.0698	6.14	9.21	12.28
.3	3.0884	6.18	9.26	12.35
.4	3.1069	6.21	9.32	12.43
—	—	—	—	—
.5	3.1256	6.25	9.38	12.50
.6	3.1443	6.29	9.43	12.58
.7	3.1630	6.33	9.49	12.65
.8	3.1818	6.36	9.55	12.73
.9	3.2007	6.40	9.60	12.80
—	—	—	—	—
34 .0	3.2196	6.44	9.66	12.88
.1	3.2385	6.48	9.71	12.95
.2	3.2576	6.51	9.77	13.03
.3	3.2766	6.55	9.83	13.11
.4	3.2958	6.59	9.89	13.18
—	—	—	—	—
.5	3.3150	6.63	9.95	13.26
.6	3.3342	6.67	10.00	13.34
.7	3.3535	6.71	10.06	13.41
.8	3.3728	6.75	10.12	13.49
.9	3.3923	6.78	10.18	13.57
—	—	—	—	—
35 .0	3.4117	6.82	10.24	13.65
.1	3.4312	6.86	10.29	13.73
.2	3.4508	6.90	10.35	13.80
.3	3.4705	6.94	10.41	13.88
.4	3.4902	6.98	10.47	13.96
—	—	—	—	—
.5	3.5099	7.02	10.53	14.04
.6	3.5297	7.06	10.59	14.12
.7	3.5496	7.10	10.65	14.20
.8	3.5695	7.14	10.71	14.28
.9	3.5894	7.18	10.77	14.36

The Young GAUGER'S Instructor. 173

in ALE GALLONS.				
Depth.				
5	6	7	8	9
14.20	17.11	19.96	22.82	25.67
14.35	17.22	20.09	22.96	25.83
14.44	17.32	20.21	23.10	25.98
14.53	17.43	20.34	23.24	26.15
14.62	17.54	20.46	23.39	26.31
-----	-----	-----	-----	-----
14.71	17.65	20.59	23.53	26.48
14.80	17.76	20.72	23.68	26.64
14.89	17.87	20.85	23.82	26.80
14.98	17.98	20.97	23.97	26.97
15.07	18.09	21.10	24.11	27.13
-----	-----	-----	-----	-----
15.17	18.20	21.23	24.26	27.30
15.26	18.31	21.36	24.41	27.46
15.35	18.42	21.49	24.56	27.63
15.44	18.53	21.62	24.71	27.75
15.53	18.64	21.75	24.85	27.96
-----	-----	-----	-----	-----
15.63	18.75	21.88	25.00	28.13
15.72	18.86	22.01	25.15	28.30
15.82	18.98	22.14	25.30	28.47
15.91	19.09	22.27	25.45	28.63
16.00	19.20	22.40	25.60	28.80
-----	-----	-----	-----	-----
16.10	19.32	22.54	25.75	28.97
16.19	19.43	22.67	25.90	29.14
16.29	19.54	22.80	26.06	29.32
16.38	19.66	22.93	26.21	29.49
16.48	19.77	23.07	26.36	29.66
-----	-----	-----	-----	-----
16.58	19.89	23.21	26.52	29.84
16.67	20.00	23.34	26.67	30.01
16.77	20.12	23.47	26.82	30.18
16.86	20.23	23.61	26.98	30.35
16.96	20.35	23.75	27.14	30.53
-----	-----	-----	-----	-----
17.06	20.47	23.88	27.29	30.70
17.16	20.59	24.01	27.45	30.88
17.25	20.70	24.16	27.61	31.06
17.35	20.82	24.29	27.76	31.24
17.45	20.94	24.43	27.92	31.41
-----	-----	-----	-----	-----
17.55	21.06	24.57	28.08	31.59
17.65	21.18	24.71	28.24	31.77
17.75	21.30	24.85	28.39	31.95
17.85	21.42	24.99	28.55	32.13
17.95	21.53	25.12	28.71	32.30

Areas

174. *The Young GAUGER'S Instructor.*

Diam. in Inch.	Areas of Circles and Contents of Cylinders			
	Depth.			
	1	2	3	4
36.0	3.6095	7.22	10.83	14.44
.1	3.6295	7.26	10.89	14.52
.2	3.6497	7.30	10.95	14.60
.3	3.6699	7.34	11.01	14.68
.4	3.6901	7.38	11.07	14.76
.5	3.7104	7.42	11.13	14.84
.6	3.7308	7.46	11.19	14.92
.7	3.7512	7.50	11.25	15.00
.8	3.7717	7.54	11.32	15.09
.9	3.7922	7.58	11.38	15.17
37.0	3.8128	7.63	11.44	15.25
.1	3.8334	7.67	11.50	15.33
.2	3.8541	7.71	11.56	15.42
.3	3.8749	7.75	11.62	15.50
.4	3.8957	7.79	11.69	15.58
.5	3.9165	7.83	11.75	15.67
.6	3.9374	7.87	11.81	15.75
.7	3.9584	7.92	11.88	15.83
.8	3.9794	7.96	11.94	15.92
.9	4.0005	8.00	12.00	16.00
38.0	4.0217	8.04	12.06	16.09
.1	4.0428	8.09	12.13	16.17
.2	4.0641	8.13	12.19	16.26
.3	4.0854	8.17	12.26	16.34
.4	4.1068	8.21	12.32	16.43
.5	4.1282	8.26	12.38	16.51
.6	4.1496	8.30	12.45	16.60
.7	4.1712	8.34	12.51	16.68
.8	4.1927	8.39	12.58	16.77
.9	4.2144	8.43	12.64	16.86
39.0	4.2361	8.47	12.71	16.94
.1	4.2578	8.52	12.77	17.03
.2	4.2796	8.56	12.84	17.12
.3	4.3015	8.60	12.90	17.20
.4	4.3234	8.65	12.97	17.29
.5	4.3454	8.69	13.04	17.38
.6	4.3674	8.73	13.10	17.47
.7	4.3895	8.78	13.17	17.56
.8	4.4117	8.82	13.24	17.65
.9	4.4338	8.87	13.30	17.74

The Young GAUGER's Instructor. 175

in ALE GALLONS.				
Depth.				
5	6	7	8	9
18.05	21.66	25.27	28.87	32.48
18.15	21.78	25.41	29.03	32.66
18.25	21.90	25.55	29.19	32.84
18.35	22.02	25.69	29.36	33.03
18.45	22.14	25.83	29.52	33.21
---	---	---	---	---
18.55	22.26	25.97	29.68	33.39
18.65	22.39	26.11	29.85	33.58
18.76	22.51	26.26	30.01	33.76
18.86	22.63	26.40	30.17	33.94
18.96	22.75	26.55	30.34	33.13
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19.07	22.88	26.69	30.51	34.32
19.17	23.00	26.83	30.67	34.50
19.27	23.12	26.98	30.83	34.69
19.37	23.25	27.12	31.00	34.87
19.48	23.37	27.27	31.17	35.06
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19.58	23.50	27.41	31.33	35.25
19.69	23.62	27.56	31.50	35.43
19.79	23.75	27.71	31.66	35.62
19.90	23.87	27.86	31.83	35.81
20.00	24.00	28.00	32.00	36.00
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20.11	24.13	28.15	32.17	36.19
20.21	24.26	28.30	32.34	36.39
20.32	24.38	28.45	32.51	36.58
20.43	24.51	28.60	32.68	36.77
20.53	24.64	28.75	32.85	36.96
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20.64	24.77	28.90	33.02	37.15
20.75	24.90	29.05	33.19	37.34
20.86	25.03	29.20	33.37	37.54
20.96	25.16	29.35	33.54	37.73
21.07	25.29	29.50	33.71	37.93
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21.18	25.42	29.65	33.89	38.12
21.29	25.55	29.80	34.06	38.32
21.40	25.68	29.96	34.23	38.51
21.51	25.81	30.11	34.41	38.71
21.62	25.94	30.26	34.58	38.91
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21.73	26.07	30.42	34.76	39.11
21.84	26.20	30.57	34.94	39.31
21.95	26.33	30.73	35.11	39.51
22.06	26.47	30.89	35.30	39.71
22.17	26.60	31.04	35.47	39.90

Areas

176 *The Young GAUGER'S Instructor.*

Diam. in Inch.	Areas of Circles and Contents of Cylinders			
	Depth.			
	1	2	3	4
40.0	4.4562	8.91	13.37	17.82
.1	4.4785	8.96	13.43	17.91
.2	4.5008	9.00	13.50	18.00
.3	4.5233	9.05	13.57	18.09
.4	4.5457	9.09	13.64	18.18
.5	4.5683	9.14	13.71	18.27
.6	4.5908	9.18	13.77	18.36
.7	4.6135	9.23	13.84	18.45
.8	4.6362	9.27	13.91	18.54
.9	4.6589	9.32	13.98	18.64
41.0	4.6818	9.36	14.05	18.73
.1	4.7046	9.41	14.11	18.82
.2	4.7275	9.45	14.18	18.91
.3	4.7505	9.50	14.25	19.00
.4	4.7735	9.55	14.32	19.09
.5	4.7966	9.59	14.39	19.19
.6	4.8198	9.64	14.46	19.28
.7	4.8430	9.69	14.53	19.37
.8	4.8662	9.73	14.60	19.46
.9	4.8895	9.78	14.67	19.56
42.0	4.9129	9.83	14.74	19.65
.1	4.9363	9.87	14.81	19.74
.2	4.9598	9.92	14.88	19.84
.3	4.9834	9.97	14.95	19.93
.4	5.0069	10.01	15.02	20.03
.5	5.0306	10.06	15.09	20.12
.6	5.0543	10.11	15.16	20.22
.7	5.0780	10.16	15.23	20.31
.8	5.1019	10.20	15.31	20.41
.9	5.1257	10.25	15.38	20.50
43.0	5.1496	10.30	15.45	20.60
.1	5.1736	10.35	15.52	20.69
.2	5.1977	10.40	15.59	20.79
.3	5.2218	10.44	15.67	20.89
.4	5.2459	10.49	15.74	20.98
.5	5.2701	10.54	15.81	21.08
.6	5.2944	10.59	15.88	21.18
.7	5.3187	10.64	15.96	21.27
.8	5.3430	10.69	16.03	21.37
.9	5.3675	10.73	16.10	21.47

The Young GAUGER's Instructor. 177

in ALE GALLONS.				
Depth.				
5	6	7	8	9
22.28	26.74	31.19	35.65	40.11
22.39	26.87	31.35	35.83	40.31
22.50	27.00	31.51	36.01	40.51
22.62	27.14	31.66	36.19	40.71
22.73	27.28	31.82	36.27	40.91
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22.84	27.41	31.98	36.55	41.11
22.95	27.55	32.14	36.73	41.32
23.07	27.68	32.29	36.91	41.52
23.18	27.82	32.45	37.09	41.72
23.30	27.95	32.61	37.27	41.93
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23.41	28.09	32.77	37.45	42.14
23.52	28.22	32.93	37.64	42.34
23.64	28.36	33.09	37.82	42.54
23.75	28.50	33.25	38.00	42.75
23.87	28.64	33.41	38.19	42.96
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23.98	28.78	33.57	38.37	43.17
24.10	28.92	33.74	38.56	43.38
24.22	29.06	33.90	38.74	43.59
24.33	29.20	34.06	38.93	43.80
24.45	29.34	34.23	39.11	44.01
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24.56	29.48	34.39	39.30	44.22
24.68	29.62	34.55	39.49	44.43
24.80	29.76	34.72	39.68	44.64
24.92	29.90	34.88	39.87	44.85
25.03	30.04	35.05	40.06	45.06
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25.15	30.18	35.21	40.24	45.28
25.27	30.33	35.38	40.43	45.49
25.39	30.47	35.55	40.62	45.70
25.51	30.61	35.71	40.82	45.92
25.63	30.75	35.88	41.01	46.13
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25.75	30.90	36.05	41.20	46.35
25.87	31.04	36.22	41.39	46.56
25.99	31.19	36.38	41.58	46.70
26.11	31.33	36.55	41.77	47.00
26.23	31.48	36.72	41.97	47.21
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26.35	31.62	36.89	42.16	47.43
26.47	31.77	37.06	42.36	47.65
26.59	31.91	37.23	42.55	47.87
26.71	32.06	37.40	42.74	48.09
26.84	32.20	37.57	42.94	48.31

A a

Areas

Diam. in Inch.	Areas of Circles and Contents of Cylinders			
	Depth.			
	1	2	3	4
44 0	5.3920	10.78	16.18	21.57
.1	5.4165	10.83	16.25	21.67
.2	5.4411	10.88	16.32	21.76
.3	5.4657	10.93	16.40	21.86
.4	5.4904	10.98	16.47	21.96
—	—	—	—	—
.5	5.5152	11.03	16.55	22.06
.6	5.5400	11.08	16.62	22.16
.7	5.5649	11.13	16.69	22.26
.8	5.5898	11.18	16.77	22.36
.9	5.6148	11.23	16.84	22.46
—	—	—	—	—
45 0	5.6398	11.28	16.92	22.56
.1	5.6649	11.33	16.99	22.66
.2	5.6901	11.38	17.07	22.76
.3	5.7153	11.43	17.15	22.86
.4	5.7405	11.48	17.22	22.96
—	—	—	—	—
.5	5.7659	11.53	17.30	23.06
.6	5.7912	11.58	17.37	23.16
.7	5.8167	11.63	17.45	23.27
.8	5.8421	11.68	17.53	23.37
.9	5.8677	11.74	17.60	23.47
—	—	—	—	—
46 0	5.8933	11.79	17.68	23.57
.1	5.9189	11.84	17.76	23.68
.2	5.9446	11.89	17.83	23.78
.3	5.9704	11.94	17.91	23.88
.4	5.9962	11.99	17.99	23.98
—	—	—	—	—
.5	6.0221	12.04	18.07	24.09
.6	6.0480	12.10	18.14	24.19
.7	6.0740	12.15	18.22	24.30
.8	6.1000	12.20	18.30	24.40
.9	6.1261	12.25	18.38	24.50
—	—	—	—	—
47 0	6.1523	12.30	18.46	24.61
.1	6.1785	12.36	18.54	24.71
.2	6.2048	12.41	18.61	24.82
.3	6.2311	12.46	18.69	24.92
.4	6.2575	12.52	18.77	25.03
—	—	—	—	—
.5	6.2839	12.57	18.85	25.14
.6	6.3104	12.62	18.93	25.24
.7	6.3369	12.67	19.01	25.35
.8	6.3635	12.73	19.09	25.45
.9	6.3902	12.78	19.17	25.56

The Young GAUGER's Instructor. 179

in ALE GALLONS.				
Depth.				
5	6	7	8	9
20.90	32.35	37.74	43.14	48.53
27.08	32.50	37.92	43.33	48.75
27.21	32.65	38.09	43.53	48.97
27.33	32.79	38.26	43.73	49.19
27.45	32.94	38.43	43.92	49.41
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27.58	33.09	38.61	44.12	49.64
27.70	33.24	38.78	44.32	49.86
27.82	33.39	38.95	44.52	50.08
27.95	33.54	39.13	44.72	50.31
28.07	33.69	39.30	44.92	50.53
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28.20	33.84	39.48	45.12	50.76
28.32	33.99	39.65	45.32	50.98
28.45	34.14	39.83	45.52	51.21
28.58	34.29	40.01	45.72	51.44
28.70	34.44	40.18	45.92	51.66
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28.83	34.60	40.36	46.13	51.89
28.96	34.75	40.54	46.33	52.12
29.08	34.90	40.72	46.53	52.35
29.21	35.05	40.89	46.74	52.58
29.34	35.21	41.07	46.94	52.81
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29.47	35.36	41.25	47.15	53.04
29.59	35.51	41.43	47.35	53.27
29.72	35.67	41.61	47.56	53.50
29.85	35.82	41.79	47.76	53.73
29.98	35.98	41.97	47.97	53.97
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30.11	36.13	42.15	48.18	54.20
30.24	36.29	42.34	48.38	54.43
30.37	36.44	42.52	48.59	54.67
30.50	36.60	42.70	48.80	54.90
30.63	36.76	42.88	49.01	55.13
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30.76	36.91	43.07	49.22	55.37
30.89	37.07	43.25	49.43	55.60
31.03	37.23	43.44	49.64	55.84
31.16	37.39	43.62	49.85	56.08
31.29	37.55	43.81	50.06	56.32
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31.42	37.70	43.99	50.27	56.56
31.55	37.86	44.17	50.48	56.79
31.68	38.02	44.36	50.70	57.03
31.82	38.18	44.54	50.91	57.27
31.95	38.34	44.73	51.12	57.51

180 *The Young GAUGER's Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
48 .0	6.4169	12.83	19.25	25.67
.1	6.4436	12.89	19.33	25.78
.2	6.4705	12.94	19.41	25.88
.3	6.4973	12.99	19.49	25.99
.4	6.5243	13.05	19.57	26.10
.5	6.5513	13.10	19.65	26.20
.6	6.5783	13.16	19.74	26.31
.7	6.6054	13.21	19.82	26.42
.8	6.6325	13.27	19.90	26.53
.9	6.6600	13.32	19.98	26.64
49 .0	6.6870	13.37	20.06	26.75
.1	6.7143	13.43	20.14	26.86
.2	6.7417	13.48	20.23	26.97
.3	6.7692	13.54	20.31	27.08
.4	6.7966	13.59	20.39	27.19
.5	6.8242	13.65	20.47	27.30
.6	6.8518	13.70	20.56	27.41
.7	6.8794	13.76	20.64	27.52
.8	6.9072	13.81	20.72	27.63
.9	6.9349	13.87	20.81	27.74
50 .0	6.9628	13.93	20.89	27.85
.1	6.9906	13.98	20.97	27.96
.2	7.0186	14.04	21.06	28.08
.3	7.0466	14.09	21.14	28.19
.4	7.0746	14.15	21.22	28.30
.5	7.1027	14.21	21.31	28.41
.6	7.1309	14.26	21.39	28.52
.7	7.1591	14.32	21.48	28.64
.8	7.1873	14.38	21.56	28.75
.9	7.2157	14.43	21.65	28.86
51 .0	7.2440	14.49	21.73	28.98
.1	7.2725	14.55	21.82	29.09
.2	7.3010	14.60	21.90	29.20
.3	7.3295	14.66	21.99	29.32
.4	7.3581	14.72	22.07	29.43
.5	7.3868	14.77	22.16	29.55
.6	7.4155	14.83	22.25	29.66
.7	7.4443	14.89	22.33	29.78
.8	7.4731	14.95	22.42	29.89
.9	7.5020	15.00	22.51	30.01

The Young GAUGER's Instructor. 181

in ALE GALLONS.				
Depth.				
5	6	7	8	9
32.08	38.50	44.92	51.34	57.75
32.22	38.66	45.11	51.55	58.00
32.35	38.82	45.29	51.76	58.23
32.48	38.98	45.48	51.98	58.48
32.62	39.15	45.67	52.19	58.72

32.76	39.31	45.86	52.41	58.96
32.89	39.47	46.05	52.63	59.20
33.03	39.63	46.24	52.84	59.45
33.16	39.79	46.43	53.06	59.69
33.30	39.96	46.62	53.28	59.94

33.43	40.12	46.81	53.50	60.18
33.57	40.29	47.00	53.71	60.43
33.71	40.45	47.19	53.93	60.68
33.85	40.61	47.38	54.15	60.92
33.99	40.78	47.58	54.37	61.17

34.12	40.95	47.77	54.59	61.42
34.26	41.11	47.96	54.81	61.67
34.40	41.28	48.15	55.03	61.91
34.54	41.44	48.35	55.26	62.16
34.68	41.61	48.54	55.48	62.41

34.82	41.78	48.74	55.71	62.66
34.96	41.95	48.94	55.93	62.92
35.10	42.11	49.13	56.15	63.17
35.24	42.28	49.33	56.38	63.42
35.38	42.45	49.53	56.60	63.68

35.52	42.62	49.72	56.83	63.93
35.66	42.79	49.92	57.05	64.18
35.80	42.96	50.11	57.27	64.43
35.94	43.12	50.31	57.50	64.69
36.08	43.30	50.51	57.73	64.94

36.22	43.47	50.71	57.95	65.20
36.36	43.63	50.91	58.18	65.45
36.51	43.81	51.11	58.41	65.71
36.65	43.97	51.31	58.64	65.96
36.79	44.15	51.51	58.87	66.22

36.94	44.32	51.71	59.10	66.48
37.08	44.49	51.91	59.32	66.74
37.22	44.66	52.11	59.55	67.00
37.37	44.84	52.31	59.78	67.26
37.51	45.01	52.52	60.02	67.52

Areas

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
52.0	7.5309	15.06	22.59	30.12
.1	7.5599	15.12	22.68	30.24
.2	7.5890	15.18	22.77	30.36
.3	7.6181	15.24	22.86	30.47
.4	7.6472	15.29	22.94	30.59
.5	7.6764	15.35	23.03	30.71
.6	7.7057	15.41	23.12	30.83
.7	7.7350	15.47	23.21	30.94
.8	7.7644	15.53	23.29	31.06
.9	7.7939	15.59	23.38	31.18
53.0	7.8233	15.65	23.47	31.29
.1	7.8529	15.71	23.56	31.41
.2	7.8825	15.76	23.65	31.53
.3	7.9122	15.82	23.74	31.65
.4	7.9419	15.88	23.83	31.77
.5	7.9717	15.94	23.92	31.89
.6	8.0015	16.00	24.00	32.01
.7	8.0314	16.06	24.09	32.13
.8	8.0613	16.12	24.18	32.24
.9	8.0913	16.18	24.27	32.36
54.0	8.1214	16.24	24.36	32.49
.1	8.1515	16.30	24.45	32.61
.2	8.1816	16.36	24.54	32.73
.3	8.2118	16.42	24.64	32.85
.4	8.2421	16.48	24.73	32.97
.5	8.2724	16.54	24.82	33.09
.6	8.3028	16.61	24.91	33.21
.7	8.3333	16.67	25.00	33.33
.8	8.3638	16.72	25.09	33.46
.9	8.3943	16.79	25.18	33.58
55.0	8.4249	16.85	25.27	33.70
.1	8.4556	16.91	25.37	33.82
.2	8.4863	16.97	25.46	33.95
.3	8.5171	17.03	25.55	34.07
.4	8.5479	17.10	25.64	34.19
.5	8.5788	17.16	25.74	34.32
.6	8.6097	17.22	25.83	34.44
.7	8.6407	17.28	25.92	34.56
.8	8.6718	17.34	26.01	34.68
.9	8.7029	17.40	26.11	34.81

The Young GAUGER's Instructor. 183

in ALE GALLONS.				
Depth.				
5	6	7	8	9
37.66	45.19	52.72	60.25	67.78
37.80	45.36	52.92	60.48	68.04
37.95	45.53	53.12	60.71	68.30
38.09	45.71	53.33	60.95	68.56
38.24	45.88	53.53	61.18	68.82
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38.38	46.06	53.73	61.41	69.09
38.53	46.24	53.94	61.65	69.36
38.68	46.41	54.15	61.88	69.62
38.82	46.59	54.35	62.11	69.88
38.97	46.76	54.56	62.35	70.15
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39.12	46.94	54.76	62.59	70.41
39.27	47.12	54.97	62.82	70.68
39.41	47.29	55.18	63.06	70.94
39.56	47.47	55.39	63.30	71.21
39.71	47.65	55.60	63.54	71.48
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39.86	47.83	55.81	63.78	71.75
40.01	48.01	56.01	64.01	72.01
40.16	48.19	56.22	64.25	72.28
40.31	48.37	56.43	64.49	72.55
40.46	48.55	56.64	64.73	72.82
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40.61	48.73	56.85	64.97	73.09
40.76	48.91	57.06	65.21	73.36
40.91	49.09	57.27	65.45	73.63
41.06	49.27	57.49	65.70	73.91
41.21	49.45	57.70	65.94	74.18
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41.36	49.63	57.91	66.18	74.45
41.52	49.82	58.12	66.43	74.73
41.67	50.00	58.33	66.67	75.00
41.82	50.19	58.55	66.91	75.28
41.97	50.37	58.76	67.15	75.55
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42.12	50.55	58.98	67.40	75.83
42.28	50.74	59.19	67.65	76.11
42.43	50.92	59.40	67.89	76.38
42.59	51.10	59.62	68.14	76.66
42.74	51.29	59.84	68.39	76.93
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42.90	51.48	60.05	68.63	77.21
43.05	51.66	60.27	68.88	77.49
43.20	51.85	60.49	69.13	77.77
43.36	52.03	60.70	69.37	78.05
43.52	52.22	60.92	69.62	78.33

Areas

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
56.0	8.7341	17.47	26.20	34.94
.1	8.7653	17.53	26.30	35.06
.2	8.7966	17.59	26.39	35.19
.3	8.8279	17.66	26.48	35.31
.4	8.8593	17.72	26.58	35.44
—	—	—	—	—
.5	8.8907	17.78	26.67	35.56
.6	8.9222	17.84	26.77	35.69
.7	8.9538	17.91	26.86	35.82
.8	8.9854	17.97	26.96	35.94
.9	9.0171	18.03	27.05	36.07
—	—	—	—	—
57.0	9.0489	18.10	27.15	36.20
.1	9.0806	18.16	27.24	36.32
.2	9.1124	18.22	27.34	36.45
.3	9.1443	18.29	27.43	36.58
.4	9.1762	18.35	27.53	36.71
—	—	—	—	—
.5	9.2082	18.42	27.62	36.83
.6	9.2403	18.48	27.72	36.96
.7	9.2724	18.54	27.82	37.09
.8	9.3046	18.61	27.91	37.22
.9	9.3368	18.67	28.01	37.35
—	—	—	—	—
58.0	9.3691	18.74	28.11	37.48
.1	9.4014	18.80	28.20	37.61
.2	9.4338	18.87	28.30	37.74
.3	9.4662	18.93	28.40	37.86
.4	9.4988	19.00	28.50	38.00
—	—	—	—	—
.5	9.5313	19.06	28.59	38.13
.6	9.5639	19.13	28.69	38.26
.7	9.5966	19.19	28.79	38.39
.8	9.6293	19.26	28.89	38.52
.9	9.6621	19.32	28.99	38.65
—	—	—	—	—
59.0	9.6949	19.39	29.08	38.78
.1	9.7278	19.46	29.18	38.91
.2	9.7608	19.52	29.28	39.04
.3	9.7938	19.59	29.38	39.18
.4	9.8268	19.65	29.48	39.31
—	—	—	—	—
.5	9.8600	19.72	29.58	39.44
.6	9.8931	19.79	29.68	39.57
.7	9.9263	19.85	29.78	39.71
.8	9.9596	19.92	29.88	39.84
.9	9.9930	19.99	29.98	39.97

The Young GAUGER's Instructor. 185

in ALE GALLONS.				
Depth.				
5	6	7	8	9
43.67	52.40	61.14	69.87	78.61
43.83	52.59	61.36	70.12	78.89
43.99	52.78	61.58	70.37	79.17
44.14	52.97	61.80	70.62	79.45
44.30	53.16	62.02	70.87	79.73
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44.46	53.35	62.24	71.13	80.02
44.61	53.53	62.46	71.38	80.30
44.77	53.72	62.68	71.63	80.59
44.93	53.91	62.90	71.88	80.87
45.09	54.10	63.12	72.14	81.15
---	---	---	---	---
45.25	54.29	63.34	72.39	81.44
45.40	54.48	63.56	72.65	81.73
45.56	54.67	63.79	72.90	82.01
45.72	54.87	64.01	73.15	82.30
45.88	55.06	64.23	73.41	82.59
---	---	---	---	---
46.04	55.25	64.46	73.67	82.87
46.20	55.44	64.68	73.92	83.16
46.36	55.63	64.91	74.18	83.45
46.52	55.83	65.13	74.43	83.74
46.69	56.02	65.36	74.69	84.03
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46.85	56.21	65.58	74.95	84.32
47.01	56.41	65.81	75.21	84.61
47.17	56.60	66.04	75.47	84.91
47.33	56.80	66.26	75.73	85.20
47.50	56.99	66.49	75.99	85.49
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47.66	57.19	66.72	76.25	85.78
47.82	57.38	66.95	76.51	86.08
47.98	57.58	67.17	76.77	86.37
48.15	57.77	67.40	77.03	86.66
48.31	57.97	67.63	77.30	86.96
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48.48	58.17	67.86	77.56	87.26
48.64	58.37	68.09	77.82	87.55
48.81	58.56	68.33	78.09	87.85
48.97	58.76	68.56	78.35	88.14
49.14	58.96	68.79	78.62	88.44
---	---	---	---	---
49.30	59.16	69.02	78.88	88.74
49.47	59.36	69.25	79.15	89.04
49.63	59.56	69.48	79.41	89.34
49.80	59.76	69.72	79.68	89.64
49.97	59.96	69.95	79.94	89.94

B b

Areas

186 *The Young GAUGER's Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
60 .0	10.0264	20.05	30.08	40.10
.1	10.0598	20.12	30.18	40.24
.2	10.0933	20.19	30.28	40.37
.3	10.1269	20.25	30.38	40.51
.4	10.1605	20.32	30.48	40.64
—	—	—	—	—
.5	10.1942	20.39	30.58	40.78
.6	10.2279	20.46	30.68	40.91
.7	10.2617	20.52	30.78	41.05
.8	10.2955	20.59	30.89	41.18
.9	10.3294	20.66	30.99	41.32
—	—	—	—	—
61 .0	10.3634	20.73	31.09	41.45
.1	10.3974	20.79	31.19	41.59
.2	10.4314	20.86	31.29	41.72
.3	10.4655	20.93	31.40	41.86
.4	10.4997	21.00	31.50	42.00
—	—	—	—	—
.5	10.5339	21.07	31.60	42.14
.6	10.5682	21.14	31.70	42.27
.7	10.6026	21.21	31.81	42.41
.8	10.6370	21.27	31.91	42.55
.9	10.6714	21.34	32.01	42.69
—	—	—	—	—
62 .0	10.7059	21.41	32.12	42.82
.1	10.7405	21.48	32.22	42.96
.2	10.7751	21.55	32.33	43.10
.3	10.8098	21.62	32.43	43.24
.4	10.8445	21.69	32.53	43.38
—	—	—	—	—
.5	10.8793	21.76	32.64	43.52
.6	10.9141	21.83	32.74	43.66
.7	10.9490	21.90	32.85	43.80
.8	10.9840	21.97	32.95	43.94
.9	11.0190	22.04	33.06	44.08
—	—	—	—	—
63 .0	11.0541	22.11	33.16	44.22
.1	11.0892	22.18	33.27	44.36
.2	11.1244	22.25	33.37	44.50
.3	11.1596	22.32	33.48	44.64
.4	11.1949	22.39	33.58	44.78
—	—	—	—	—
.5	11.2302	22.46	33.69	44.92
.6	11.2656	22.53	33.79	45.06
.7	11.3011	22.60	33.90	45.20
.8	11.3366	22.67	34.01	45.35
.9	11.3721	22.74	34.12	45.49

The Young GAUGER's Instructor. 187

in ALE GALLONS.				
Depth.				
5	6	7	8	9
50.13	60.10	70.18	80.21	90.24
50.30	60.36	70.42	80.48	90.54
50.47	60.56	70.65	80.75	90.84
50.64	60.76	70.89	81.02	91.14
50.80	60.96	71.12	81.28	91.44
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50.97	61.17	71.36	81.55	91.75
51.14	61.37	71.60	81.82	92.05
51.31	61.57	71.83	82.09	92.36
51.48	61.77	72.07	82.36	92.66
51.65	61.98	72.31	82.63	92.96
---	---	---	---	---
51.82	62.18	72.54	82.91	93.27
51.99	62.38	72.78	83.18	93.58
52.16	62.59	73.02	83.45	93.88
52.32	62.79	73.26	83.72	94.19
52.50	63.00	73.50	84.00	94.50
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52.67	63.20	73.74	84.27	94.81
52.84	63.41	73.98	84.55	95.11
53.01	63.62	74.22	84.82	95.42
53.18	63.82	74.46	85.10	95.73
53.36	64.03	74.70	85.37	96.04
---	---	---	---	---
53.53	64.24	74.94	85.65	96.35
53.70	64.44	75.18	85.92	96.66
53.88	64.65	75.43	86.20	96.98
54.05	64.86	75.67	86.48	97.29
54.22	65.07	75.91	86.75	97.60
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54.40	65.28	76.15	87.03	97.91
54.57	65.48	76.40	87.31	98.23
54.74	65.69	76.64	87.59	98.54
54.92	65.90	76.89	87.87	98.86
55.09	66.11	77.13	88.15	99.17
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55.27	66.32	77.38	88.43	99.49
55.45	66.53	77.62	88.71	99.80
55.62	66.75	77.87	88.99	100.12
55.80	66.96	78.12	89.28	100.44
55.97	67.17	78.37	89.56	100.75
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56.15	67.38	78.61	89.84	101.07
56.33	67.59	78.86	90.13	101.39
56.51	67.81	79.11	90.41	101.71
56.69	68.02	79.36	90.70	102.03
56.86	68.23	79.60	90.98	102.35

188 *The Young GAUGER's Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
64 .0	11.4078	22.82	34.22	45.63
.1	11.4434	22.89	34.33	45.77
.2	11.4792	22.96	34.44	45.92
.3	11.5150	23.03	34.54	46.06
.4	11.5508	23.10	34.65	46.20
.5	11.5867	23.17	34.76	46.35
.6	11.6227	23.25	34.84	46.49
.7	11.6587	23.32	34.98	46.63
.8	11.6947	23.39	35.08	46.78
.9	11.7309	23.46	35.19	46.92
65 .0	11.7670	23.53	35.30	47.07
.1	11.8033	23.61	35.41	47.21
.2	11.8396	23.68	35.52	47.36
.3	11.8759	23.75	35.63	47.50
.4	11.9123	23.82	35.74	47.65
.5	11.9488	23.90	35.85	47.80
.6	11.9853	23.97	35.96	47.94
.7	12.0219	24.04	36.07	48.09
.8	12.0585	24.12	36.18	48.23
.9	12.0952	24.19	36.29	48.38
66 .0	12.1319	24.26	36.40	48.53
.1	12.1687	24.34	36.51	48.67
.2	12.2055	24.41	36.62	48.82
.3	12.2424	24.48	36.73	48.97
.4	12.2794	24.56	36.84	49.12
.5	12.3164	24.63	36.95	49.27
.6	12.3535	24.71	37.06	49.41
.7	12.3906	24.78	37.17	49.56
.8	12.4278	24.86	37.28	49.71
.9	12.4650	24.93	37.40	49.86
67 .0	12.5023	25.00	37.51	50.01
.1	12.5397	25.08	37.62	50.16
.2	12.5771	25.15	37.73	50.31
.3	12.6145	25.23	37.84	50.46
.4	12.6520	25.30	37.96	50.61
.5	12.6896	25.38	38.07	50.76
.6	12.7272	25.45	38.18	50.91
.7	12.7649	25.53	38.29	51.06
.8	12.8027	25.61	38.41	51.21
.9	12.8405	25.68	38.52	51.36

in

The Young GAUGER's Instructor. 189

in ALE GALLONS.				
Depth.				
5	6	7	8	9
57.04	68.45	79.85	91.26	102.67
57.22	68.66	80.10	91.55	102.99
57.40	68.87	80.35	91.83	103.31
57.57	69.09	80.61	92.12	103.64
57.75	69.31	80.86	92.41	103.96
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57.93	69.52	81.11	92.69	104.28
58.11	69.78	81.36	92.98	104.60
58.29	69.95	81.61	93.27	104.93
58.47	70.17	81.86	93.56	105.26
58.65	70.39	82.12	93.85	105.58
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58.84	70.60	82.37	94.14	105.90
59.02	70.82	82.62	94.43	106.23
59.20	71.04	82.88	94.72	106.56
59.38	71.26	83.13	95.01	106.88
59.56	71.47	83.39	95.30	107.21
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59.74	71.69	83.64	95.59	107.54
59.93	71.91	83.90	95.88	107.87
60.11	72.13	84.15	96.18	108.20
60.29	72.35	84.41	96.47	108.53
60.48	72.57	84.67	96.76	108.86
---	---	---	---	---
60.66	72.79	84.92	97.06	109.19
60.84	73.01	85.18	97.35	109.52
61.03	73.23	85.44	97.64	109.85
61.21	73.45	85.70	97.94	110.18
61.40	73.67	85.96	98.24	110.51
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61.58	73.90	86.21	98.53	110.85
61.77	74.12	86.47	98.83	111.18
61.95	74.34	86.73	99.12	111.52
62.14	74.57	86.99	99.42	111.85
62.33	74.79	87.26	99.72	112.19
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62.51	75.01	87.52	100.02	112.52
62.70	75.24	87.78	100.32	112.86
62.89	75.46	88.04	100.62	113.19
63.07	75.69	88.30	100.92	113.53
63.26	75.91	88.56	101.22	113.87
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63.45	76.14	88.82	101.52	114.21
63.64	76.36	89.09	101.82	114.54
63.82	76.59	89.35	102.12	114.88
64.01	76.82	89.62	102.42	115.22
64.20	77.04	89.88	102.72	115.56

Areas

190 *The Young GAUGER'S Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
68.0	12.8783	25.76	38.63	51.51
.1	12.9162	25.83	38.75	51.66
.2	12.9542	25.91	38.86	51.82
.3	12.9922	25.98	38.98	51.97
.4	13.0303	26.06	39.09	52.12
.5	13.0684	26.14	39.21	52.27
.6	13.1066	26.21	39.32	52.43
.7	13.1448	26.29	39.43	52.58
.8	13.1831	26.37	39.55	52.73
.9	13.2215	26.44	39.66	52.89
69.0	13.2599	26.52	39.78	53.04
.1	13.2983	26.60	39.89	53.19
.2	13.3368	26.67	40.01	53.35
.3	13.3754	26.75	40.13	53.50
.4	13.4140	26.83	40.24	53.66
.5	13.4527	26.91	40.36	53.81
.6	13.4915	26.98	40.47	53.96
.7	13.5303	27.06	40.59	54.12
.8	13.5691	27.14	40.71	54.28
.9	13.6080	27.22	40.82	54.43
70.0	13.6470	27.29	40.94	54.59
.1	13.6860	27.37	41.06	54.74
.2	13.7251	27.45	41.18	54.90
.3	13.7742	27.53	41.29	55.06
.4	13.8034	27.61	41.41	55.21
.5	13.8426	27.69	41.53	55.37
.6	13.8819	27.76	41.65	55.53
.7	13.9213	27.84	41.76	55.69
.8	13.9607	27.92	41.88	55.84
.9	14.0002	28.00	42.00	56.00
71.0	14.0397	28.08	42.12	56.16
.1	14.0793	28.16	42.24	56.32
.2	14.1189	28.24	42.36	56.48
.3	14.1586	28.32	42.48	56.63
.4	14.1983	28.40	42.59	56.79
.5	14.2381	28.48	42.71	56.95
.6	14.2780	28.56	42.83	57.11
.7	14.3179	28.64	42.95	57.27
.8	14.3579	28.72	43.07	57.43
.9	14.3979	28.80	43.19	57.59

The Young GAUGER'S Instructor. 191

in ALL GALLONS.				
Depth.				
5	6	7	8	9
64.39	77.27	90.15	103.03	115.90
64.58	77.50	90.41	103.33	116.24
64.77	77.73	90.68	103.63	116.59
64.96	77.95	90.95	103.94	116.93
65.15	78.18	91.21	104.24	117.27
65.34	78.41	91.48	104.55	117.62
65.53	78.64	91.75	104.85	117.96
65.72	78.87	92.01	105.16	118.30
65.92	79.10	92.28	105.46	118.65
66.11	79.33	92.55	105.77	118.99
66.30	79.56	92.82	106.08	119.34
66.49	79.79	93.09	106.39	119.68
66.68	80.02	93.36	106.69	120.03
66.88	80.25	93.63	107.00	120.38
67.07	80.48	93.90	107.31	120.73
67.26	80.72	94.17	107.62	121.07
67.46	80.95	94.44	107.93	121.42
67.65	81.18	94.71	108.24	121.77
67.85	81.41	94.98	108.55	122.12
68.04	81.65	95.26	108.86	122.47
68.24	81.88	95.53	109.18	122.82
68.43	82.12	95.80	109.49	123.17
68.63	82.35	96.08	109.80	123.53
68.82	82.59	96.35	110.11	123.88
69.02	82.82	96.62	110.43	124.23
69.21	83.05	96.89	110.74	124.58
69.41	83.29	97.17	111.06	124.94
69.61	83.53	97.45	111.37	125.29
69.80	83.77	97.73	111.69	125.65
70.00	84.00	98.00	112.00	126.00
70.20	84.24	98.28	112.32	126.36
70.40	84.48	98.56	112.63	126.71
70.59	84.71	98.83	112.95	127.07
70.79	84.95	99.11	113.27	127.43
70.99	85.19	99.39	113.59	127.78
71.19	85.43	99.67	113.90	128.14
71.39	85.67	99.95	114.22	128.50
71.59	85.91	100.23	114.54	128.86
71.79	86.15	100.51	114.86	129.22
71.99	86.39	100.79	115.18	129.58

Areas

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
72 .0	14.4380	28.88	43.31	57.75
.1	14.4781	28.96	43.43	57.91
.2	14.5183	29.04	43.55	58.07
.3	14.5585	29.12	43.68	58.23
.4	14.5988	29.20	43.80	58.40

.5	14.6392	29.28	43.92	58.56
.6	14.6796	29.36	44.04	58.72
.7	14.7201	29.44	44.16	58.88
.8	14.7606	29.52	44.28	59.04
.9	14.8012	29.60	44.40	59.20

73 .0	14.8418	29.68	44.53	59.37
.1	14.8825	29.76	44.65	59.53
.2	14.9232	29.85	44.77	59.69
.3	14.9640	29.93	44.89	59.86
.4	15.0049	30.01	45.01	60.02

.5	15.0458	30.09	45.14	60.18
.6	15.0868	30.17	45.26	60.35
.7	15.1278	30.26	45.38	60.51
.8	15.1689	30.34	45.51	60.68
.9	15.2100	30.42	45.63	60.84

74 .0	15.2512	30.50	45.75	61.00
.1	15.2925	30.58	45.88	61.17
.2	15.3338	30.67	46.00	61.34
.3	15.3751	30.75	46.13	61.50
.4	15.4165	30.83	46.25	61.67

.5	15.4580	30.92	46.37	61.83
.6	15.4996	31.00	46.50	62.00
.7	15.5411	31.08	46.62	62.16
.8	15.5827	31.17	46.75	62.33
.9	15.6244	31.25	46.87	62.50

75 .0	15.6662	31.33	47.00	62.66
.1	15.7080	31.42	47.12	62.83
.2	15.7499	31.50	47.25	63.00
.3	15.7918	31.58	47.38	63.17
.4	15.8337	31.67	47.50	63.33

.5	15.8758	31.75	47.63	63.50
.6	15.9178	31.84	47.75	63.67
.7	15.9600	31.92	47.88	63.84
.8	16.0022	32.00	48.01	64.01
.9	16.0444	32.09	48.13	64.18

The Young GAUGER's Instructor. 193

<i>in ALE GALLONS.</i>				
Depth.				
5	6	7	8	9
72.19	86.63	101.07	115.50	129.94
72.39	86.87	101.35	115.82	130.30
72.59	87.11	101.63	116.15	130.67
72.79	87.35	101.91	116.47	131.03
72.99	87.59	102.19	116.79	131.39
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73.20	87.83	102.47	117.11	131.75
73.40	88.07	102.76	117.43	132.11
73.60	88.32	103.04	117.76	132.48
73.80	88.56	103.32	118.08	132.84
74.01	88.81	103.61	118.41	133.21
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74.21	89.05	103.89	118.73	133.58
74.41	89.29	104.17	119.06	133.94
74.62	89.54	104.46	119.39	134.31
74.82	89.78	104.75	119.71	134.68
75.02	90.03	105.03	120.04	135.04
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75.23	90.28	105.32	120.37	135.41
75.43	90.52	105.61	120.69	135.78
75.64	90.77	105.89	121.02	136.15
75.84	91.01	106.18	121.35	136.52
76.05	91.26	106.47	121.68	136.89
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76.26	91.51	106.76	122.01	137.26
76.46	91.75	107.05	122.34	137.63
76.67	92.00	107.34	123.67	138.00
76.88	92.25	107.63	123.00	138.38
77.08	92.50	107.92	123.33	138.75
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77.29	92.75	108.21	123.66	139.12
77.50	93.00	108.50	124.00	139.50
77.71	93.25	108.79	124.33	139.87
77.91	93.50	109.08	124.66	140.24
78.12	93.75	109.37	124.99	140.62
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78.33	94.00	109.66	125.33	141.00
78.54	94.25	109.96	125.66	141.37
78.75	94.50	110.25	126.00	141.75
78.96	94.75	110.54	126.33	142.13
79.17	95.00	110.84	126.67	142.50
-----	-----	-----	-----	-----
79.38	95.25	111.13	127.01	142.88
79.59	95.51	111.42	127.34	143.26
79.80	95.76	111.72	127.68	143.64
80.01	96.01	112.01	128.02	144.02
80.22	96.26	112.31	128.35	144.40

C c

Areas

194 *The Young GAUGER's Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
76.0	16.0867	32.17	48.26	64.35
.1	16.1291	32.26	48.39	64.52
.2	16.1715	32.34	48.51	64.69
.3	16.2140	32.43	48.64	64.86
.4	16.2565	32.51	48.77	65.03
.5	16.2991	32.60	48.90	65.20
.6	16.3417	32.68	49.03	65.37
.7	16.3844	32.77	49.15	65.54
.8	16.4272	32.85	49.28	65.71
.9	16.4700	32.94	49.41	65.88
77.0	16.5129	33.03	49.54	66.05
.1	16.5558	33.11	49.67	66.22
.2	16.5988	33.20	49.80	66.40
.3	16.6418	33.28	49.93	66.57
.4	16.6849	33.37	50.06	66.74
.5	16.7280	33.46	50.18	66.91
.6	16.7712	33.54	50.31	67.08
.7	16.8145	33.63	50.44	67.26
.8	16.8578	33.72	50.57	67.43
.9	16.9011	33.80	50.70	67.60
78.0	16.9445	33.89	50.83	67.78
.1	16.9880	33.98	50.96	67.95
.2	17.0316	34.06	51.09	68.13
.3	17.0751	34.15	51.23	68.30
.4	17.1188	34.24	51.36	68.48
.5	17.1625	34.32	51.49	68.65
.6	17.2062	34.41	51.62	68.82
.7	17.2500	34.50	51.75	69.00
.8	17.2939	34.59	51.88	69.18
.9	17.3378	34.68	52.01	69.35
79.0	17.3818	34.76	52.15	69.53
.1	17.4258	34.85	52.28	69.70
.2	17.4699	34.94	52.41	69.88
.3	17.5141	35.03	52.54	70.06
.4	17.5583	35.12	52.67	70.23
.5	17.6025	35.20	52.81	70.41
.6	17.6468	35.29	52.94	70.59
.7	17.6912	35.38	53.07	70.76
.8	17.7356	35.47	53.21	70.94
.9	17.7801	35.56	53.34	71.12

The Young GAUGER's Instructor. 195

in ALE GALLONS.				
Depth.				
5	6	7	8	9
80.43	96.52	112.61	128.69	144.78
80.65	96.77	112.90	129.63	145.16
80.86	97.03	113.20	129.37	145.54
81.07	97.28	113.50	129.71	145.93
81.28	97.54	113.79	130.05	146.31

81.50	97.79	114.09	130.39	146.69
81.71	98.05	114.39	130.73	147.08
81.92	98.31	114.69	131.08	147.46
82.14	98.56	114.99	131.42	147.84
82.35	98.82	115.29	131.76	148.23

82.57	99.08	115.59	132.10	148.62
82.78	99.34	115.89	132.45	149.00
83.00	99.59	116.19	132.76	149.39
83.21	99.85	116.49	133.14	149.78
83.42	100.11	116.79	133.48	150.16

83.64	100.37	117.10	133.82	150.55
83.86	100.63	117.40	134.17	150.94
84.07	100.89	117.70	134.51	151.33
84.29	101.15	118.00	134.86	151.72
84.51	101.41	118.31	135.21	152.11

84.72	101.67	118.61	135.56	152.50
84.94	101.93	118.92	135.90	152.89
85.16	102.19	119.22	136.25	153.28
85.38	102.45	119.53	136.60	153.68
85.59	102.71	119.83	136.95	154.07

85.81	102.97	120.13	137.30	154.46
86.03	103.24	120.44	137.65	154.85
86.25	103.50	120.75	138.00	155.25
86.47	103.76	121.06	138.35	155.65
86.69	104.03	121.37	138.78	156.04

86.91	104.29	121.67	139.05	156.44
87.13	104.55	121.98	139.41	156.83
87.35	104.82	122.29	139.76	157.23
87.57	105.08	122.60	140.11	157.63
87.79	105.35	122.91	140.47	158.02

88.01	105.61	123.22	140.82	158.42
88.23	105.88	123.53	141.17	158.82
88.46	106.15	123.84	141.53	159.22
88.68	106.41	124.15	141.88	159.62
88.90	106.68	124.46	142.24	160.02

196 *The Young GAUGER's Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
80 .0	17.8246	35.65	53.47	71.30
.1	17.8692	35.74	53.61	71.48
.2	17.9139	35.83	53.74	71.66
.3	17.9586	35.92	53.87	71.83
.4	18.0033	36.01	54.01	72.01
.5	18.0481	36.10	54.14	72.19
.6	18.0930	36.19	54.28	72.37
.7	18.1379	36.28	54.42	72.55
.8	18.1829	36.37	54.55	72.73
.9	18.2280	36.46	54.68	72.91
81 .0	18.2730	36.55	54.82	73.09
.1	18.3182	36.64	54.95	73.27
.2	18.3634	36.73	55.09	73.45
.3	18.4086	36.82	55.23	73.63
.4	18.4540	36.91	55.36	73.82
.5	18.4993	37.00	55.50	74.00
.6	18.5448	37.09	55.64	74.18
.7	18.5902	37.18	55.77	74.36
.8	18.6358	37.27	55.91	74.54
.9	18.6814	37.36	56.04	74.72
82 .0	18.7270	37.45	56.18	74.91
.1	18.7727	37.55	56.32	75.09
.2	18.8185	37.64	56.45	75.27
.3	18.8643	37.73	56.59	75.46
.4	18.9102	37.82	56.73	75.64
.5	18.9561	37.91	56.87	75.82
.6	19.0021	38.00	57.01	76.01
.7	19.0481	38.10	57.14	76.19
.8	19.0942	38.19	57.28	76.38
.9	19.1403	38.28	57.42	76.56
83 .0	19.1866	38.37	57.56	76.75
.1	19.2328	38.47	57.70	76.93
.2	19.2791	38.56	57.84	77.12
.3	19.3255	38.65	57.98	77.30
.4	19.3719	38.74	58.12	77.49
.5	19.4184	38.84	58.26	77.67
.6	19.4650	38.93	58.40	77.86
.7	19.5115	39.02	58.53	78.05
.8	19.5582	39.12	58.67	78.23
.9	19.6049	39.21	58.81	78.42

The Young GAUGER's Instructor. 197

<i>in</i> ALE GALLONS.				
Depth.				
5	6	7	8	9
89.12	106.95	124.77	142.59	160.42
89.35	107.22	125.08	142.95	160.82
89.57	107.48	125.40	143.31	161.23
89.79	107.75	125.71	143.67	161.63
90.02	108.02	126.02	144.03	162.03
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90.24	108.29	126.34	144.39	162.43
90.47	108.56	126.65	144.74	162.84
90.69	108.83	126.97	145.10	163.24
90.91	109.10	127.28	145.46	163.65
91.14	109.37	127.60	145.82	164.05
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91.37	109.64	127.91	146.18	164.46
91.59	109.91	128.23	146.55	164.86
91.82	110.18	128.54	146.91	165.27
92.04	110.45	128.86	147.27	165.68
92.27	110.72	129.18	147.63	166.09
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92.50	110.99	129.50	147.99	166.49
92.72	111.27	129.81	148.36	166.90
92.95	111.54	130.13	148.72	167.31
93.18	111.82	130.45	149.09	167.72
93.41	112.09	130.77	149.45	168.13
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93.64	112.36	131.09	149.82	168.54
93.87	112.64	131.41	150.18	168.95
94.09	112.91	131.73	150.55	169.37
94.32	113.18	132.05	150.91	169.78
94.55	113.46	132.37	151.28	170.19
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94.78	113.74	132.69	151.65	170.60
95.01	114.01	133.01	152.02	171.02
95.24	114.29	133.34	152.39	171.43
95.47	114.56	133.66	152.75	171.85
95.70	114.84	133.98	153.12	172.26
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95.93	115.12	134.31	153.49	172.68
96.16	115.40	134.63	153.86	173.10
96.40	115.67	134.95	154.23	173.51
96.63	115.95	135.28	154.60	173.93
96.86	116.23	135.60	154.98	174.35
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97.09	116.51	135.93	155.35	174.77
97.33	116.79	136.26	155.72	175.19
97.56	117.07	136.58	156.09	175.60
97.79	117.35	136.91	156.47	176.02
98.02	117.63	137.23	156.84	176.44

Areas

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
84 .0	19.6517	39.30	58.96	78.61
.1	19.6985	39.40	59.10	78.79
.2	19.7454	39.49	59.24	78.98
.3	19.7923	39.58	59.38	79.17
.4	19.8393	39.68	59.52	79.36
.5	19.8863	39.77	59.66	79.55
.6	19.9334	39.87	59.80	79.73
.7	19.9806	39.96	59.94	79.92
.8	20.0278	40.06	60.08	80.11
.9	20.0750	40.15	60.23	80.30
85 .0	20.1223	40.24	60.37	80.49
.1	20.1697	40.34	60.51	80.68
.2	20.2172	40.43	60.65	80.87
.3	20.2646	40.53	60.79	81.06
.4	20.3122	40.62	60.94	81.25
.5	20.3598	40.72	61.08	81.44
.6	20.4074	40.81	61.22	81.63
.7	20.4551	40.91	61.37	81.82
.8	20.5029	41.01	61.51	82.01
.9	20.5507	41.10	61.65	82.20
86 .0	20.5986	41.20	61.80	82.39
.1	20.6465	41.29	61.94	82.59
.2	20.6945	41.39	62.08	82.78
.3	20.7426	41.49	62.23	82.97
.4	20.7907	41.58	62.37	83.16
.5	20.8388	41.68	62.52	83.36
.6	20.8870	41.77	62.66	83.55
.7	20.9353	41.87	62.81	83.74
.8	20.9836	41.97	62.95	83.93
.9	20.0320	42.06	63.10	84.13
87 .0	21.0804	42.16	63.24	84.32
.1	21.1289	42.26	63.39	84.52
.2	21.1775	42.35	63.53	84.71
.3	21.2261	42.45	63.68	84.90
.4	21.2747	42.55	63.82	85.10
.5	21.3234	42.65	63.97	85.29
.6	21.3722	42.74	64.12	85.49
.7	21.4210	42.84	64.26	85.68
.8	21.4699	42.94	64.41	85.88
.9	21.5188	43.04	64.56	86.08

The Young GAUGER's Instructor. 199

in ALE GALLONS.				
Depth.				
5	6	7	8	9
98.26	117.91	137.56	157.21	176.87
98.49	118.19	137.89	157.59	177.29
98.73	118.47	138.22	157.96	177.71
98.96	118.75	138.55	158.34	178.13
99.20	119.03	138.88	158.71	178.55
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99.43	119.32	139.20	159.09	178.98
99.67	119.60	139.53	159.47	179.40
99.90	119.88	139.86	159.84	179.83
100.14	120.17	140.19	160.22	180.25
100.38	120.45	140.53	160.60	180.68
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100.61	120.73	140.86	160.98	181.10
100.85	121.02	141.19	161.36	181.53
101.09	121.30	141.52	161.74	181.95
101.32	121.59	141.85	162.12	182.38
101.56	121.87	142.18	162.50	182.81
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101.80	122.16	142.52	162.88	183.24
102.04	122.44	142.85	163.26	183.67
102.28	122.73	143.19	163.64	184.10
102.52	123.02	143.52	164.02	184.53
102.76	123.31	143.86	164.41	184.96
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102.99	123.59	144.19	164.79	185.39
103.23	123.88	144.53	165.17	185.82
103.47	124.17	144.86	165.55	186.25
103.71	124.46	145.20	165.94	186.68
103.95	124.74	145.53	166.33	187.12
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104.19	125.03	145.87	166.71	187.55
104.44	125.32	146.21	167.10	187.98
104.68	125.61	146.55	167.48	188.42
104.92	125.90	146.89	167.87	188.85
105.16	126.19	147.22	168.26	189.29
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105.40	126.48	147.56	168.64	189.72
105.65	126.77	147.90	169.03	190.16
105.89	127.06	148.24	169.42	190.60
106.13	127.36	148.58	169.81	191.03
106.38	127.65	148.92	170.20	191.47
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106.62	127.94	149.26	170.59	191.91
106.86	128.23	149.60	170.98	192.35
107.11	128.53	149.95	171.37	192.79
107.35	128.82	150.29	171.76	193.23
107.59	129.11	150.63	172.15	193.67

Areas

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
88.0	21.5678	43.14	64.70	86.27
.1	21.6169	43.23	64.85	86.47
.2	21.6660	43.33	65.00	86.66
.3	21.7151	43.43	65.15	86.86
.4	21.7643	43.53	65.29	87.06
.5	21.8136	43.63	65.44	87.25
.6	21.8629	43.73	65.59	87.45
.7	21.9123	43.82	65.74	87.65
.8	21.9617	43.92	65.89	87.85
.9	22.0112	44.02	66.03	88.04
89.0	22.0608	44.12	66.18	88.24
.1	22.1104	44.22	66.33	88.44
.2	22.1600	44.32	66.48	88.64
.3	22.2098	44.42	66.63	88.84
.4	22.2595	44.52	66.78	89.04
.5	22.3093	44.62	66.93	89.24
.6	22.3592	44.72	67.08	89.44
.7	22.4092	44.82	67.23	89.64
.8	22.4592	44.92	67.38	89.84
.9	22.5092	45.02	67.53	90.04
90.0	22.5593	45.12	67.68	90.24
.1	22.6095	45.22	67.83	90.44
.2	22.6597	45.32	67.98	90.64
.3	22.7100	45.42	68.13	90.84
.4	22.7603	45.52	68.28	91.04
.5	22.8107	45.62	68.43	91.24
.6	22.8611	45.72	68.58	91.44
.7	22.9116	45.82	68.73	91.65
.8	22.9621	45.92	68.89	91.85
.9	23.0128	46.03	69.04	92.05
91.0	23.0634	46.13	69.19	92.25
.1	23.1141	46.23	69.34	92.46
.2	23.1649	46.33	69.49	92.66
.3	23.2157	46.43	69.65	92.86
.4	23.2666	46.53	69.80	93.07
.5	23.3176	46.64	69.95	93.27
.6	23.3685	46.74	70.11	93.47
.7	23.4196	46.84	70.26	93.68
.8	23.4707	46.94	70.41	93.88
.9	23.5219	47.04	70.57	94.09

The Young GAUGER'S Instructor. 201

<i>in ALE GALLONS.</i>				
Depth.				
5	6	7	8	9
107.84	129.41	150.97	172.54	194.11
108.08	129.70	151.32	172.94	194.55
108.33	130.00	151.66	173.33	194.99
108.58	130.29	152.01	173.72	195.44
108.82	130.59	152.35	174.11	195.88
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109.07	130.88	152.70	174.51	196.32
109.31	131.18	153.04	174.90	196.77
109.56	131.47	153.39	175.30	197.21
109.81	131.77	153.73	175.69	197.66
110.06	132.07	154.08	176.09	198.10
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110.30	132.36	154.43	176.49	198.55
110.55	132.66	154.73	176.88	198.99
110.80	132.96	155.12	177.28	199.44
111.05	133.26	155.47	177.68	199.89
111.30	133.56	155.82	178.08	200.34
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111.55	133.86	156.17	178.47	200.78
111.80	134.16	156.51	178.87	201.23
112.05	134.46	156.86	179.27	201.68
112.30	134.76	157.21	179.67	202.13
112.55	135.06	157.56	180.07	202.58
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112.80	135.36	157.92	180.47	203.03
113.05	135.66	158.27	180.88	203.49
113.30	135.96	158.62	181.28	203.94
113.55	136.26	158.97	181.68	204.39
113.80	136.56	159.32	182.08	204.84
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114.05	136.86	159.67	182.49	205.30
114.31	137.17	160.03	182.89	205.75
114.56	137.47	160.38	183.29	206.20
114.81	137.77	160.73	183.70	206.66
115.06	138.08	161.09	184.10	207.12
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115.32	138.38	161.44	184.51	207.57
115.57	138.68	161.80	184.91	208.03
115.82	138.99	162.15	185.32	208.48
116.08	139.29	162.51	185.73	208.94
116.33	139.60	162.87	186.13	209.40
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116.59	139.91	163.22	186.54	209.86
116.84	140.21	163.58	186.95	210.32
117.10	140.52	163.94	187.36	210.78
117.35	140.82	164.29	187.77	211.24
117.61	141.13	164.65	188.18	211.70

D d

Areas

Diam. in Inch.	Areas of Circles and Contents of Cylinders			
	Depth.			
	1	2	3	4
92.0	23.5731	47.15	70.72	94.29
.1	23.6244	47.25	70.87	94.50
.2	23.6757	47.35	71.03	94.70
.3	23.7271	47.45	71.18	94.91
.4	23.7785	47.56	71.33	95.11
.5	23.8300	47.66	71.49	95.32
.6	23.8816	47.76	71.64	95.53
.7	23.9332	47.87	71.80	95.73
.8	23.9848	47.97	71.95	95.94
.9	24.0366	48.07	72.11	96.15
93.0	24.0883	48.18	72.26	96.35
.1	24.1402	48.28	72.42	96.56
.2	24.1920	48.38	72.58	96.77
.3	24.2440	48.49	72.73	96.98
.4	24.2960	48.59	72.89	97.18
.5	24.3480	48.70	73.04	97.39
.6	24.4001	48.80	73.20	97.60
.7	24.4523	48.90	73.36	97.81
.8	24.5045	49.01	73.51	98.02
.9	24.5568	49.11	73.67	98.23
94.0	24.6091	49.22	73.83	98.44
.1	24.6615	49.32	73.98	98.65
.2	24.7140	49.43	74.14	98.86
.3	24.7665	49.53	74.30	99.07
.4	24.8190	49.64	74.46	99.28
.5	24.8716	49.74	74.61	99.49
.6	24.9243	49.85	74.77	99.70
.7	24.9770	49.95	74.93	99.91
.8	25.0298	50.06	75.09	100.12
.9	25.0826	50.17	75.25	100.33
95.0	25.1355	50.27	75.41	100.54
.1	25.1885	50.38	75.57	100.75
.2	25.2415	50.48	75.72	100.96
.3	25.2945	50.59	75.88	101.18
.4	25.3476	50.70	76.04	101.39
.5	25.4008	50.80	76.20	101.60
.6	25.4540	50.91	76.36	101.82
.7	25.5073	51.01	76.52	102.03
.8	25.5606	51.12	76.68	102.24
.9	25.6140	51.23	76.84	102.46

in ALE GALLONS.				
Depth.				
5	6	7	8	9
117.87	141.44	165.01	188.58	212.16
118.12	141.75	165.37	189.00	212.62
118.38	142.05	165.73	189.41	213.08
118.64	142.36	166.09	189.82	213.54
118.89	142.67	166.45	190.23	214.00
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119.15	142.98	166.81	190.64	214.47
119.41	143.29	167.17	191.05	214.93
119.67	143.60	167.53	191.46	215.40
119.92	143.91	167.89	191.88	215.86
120.18	144.22	168.26	192.29	216.32
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120.44	144.53	168.62	192.70	216.79
120.70	144.84	168.98	193.12	217.26
120.96	145.15	169.34	193.54	217.73
121.22	145.46	169.71	193.95	218.20
121.48	145.78	170.07	194.37	218.66
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121.74	146.09	170.44	194.78	219.13
122.00	146.40	170.80	195.20	219.60
122.26	146.71	171.17	195.62	220.07
122.52	147.03	171.53	196.07	220.54
122.78	147.34	171.90	196.45	221.01
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123.05	147.65	172.26	196.87	221.48
123.31	147.97	172.63	197.29	221.95
123.57	148.28	173.00	197.71	222.43
123.83	148.60	173.37	198.13	222.90
124.10	148.91	173.73	198.55	223.37
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124.36	149.23	174.10	198.97	223.84
124.62	149.55	174.47	199.39	224.32
124.89	149.86	174.84	199.82	224.79
125.15	150.18	175.21	200.24	225.27
125.41	150.50	175.58	200.66	225.74
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125.68	150.81	175.95	201.08	226.22
125.94	151.13	176.32	201.51	226.70
126.21	151.45	176.69	201.93	227.17
126.47	151.77	177.06	202.36	227.65
126.74	152.09	177.43	202.78	228.13
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127.00	152.40	177.81	203.21	228.61
127.27	152.72	178.18	203.63	229.09
127.54	153.04	178.55	204.06	229.57
127.80	153.36	178.92	204.48	230.05
128.07	153.68	179.30	204.91	230.53

204 *The Young GAUGER'S Instructor.*

Diam. in Inch.	Areas of Circles and Contents of Cylinders			
	Depth.			
	1	2	3	4
96.0	25.6675	51.33	77.00	102.67
.1	25.7210	51.44	77.16	102.88
.2	25.7345	51.55	77.32	103.10
.3	25.8282	51.66	77.48	103.31
.4	25.8818	51.76	77.65	103.53
.5	25.9355	51.87	77.81	103.74
.6	25.9893	51.98	77.97	103.96
.7	26.0432	52.09	78.13	104.17
.8	26.0971	52.19	78.29	104.39
.9	26.1510	52.30	78.45	104.60
97.0	26.2050	52.41	78.62	104.82
.1	26.2591	52.52	78.78	105.04
.2	26.3132	52.63	78.94	105.25
.3	26.3673	52.73	79.10	105.47
.4	26.4216	52.84	79.26	105.69
.5	26.4759	52.95	79.43	105.90
.6	26.5302	53.06	79.59	106.12
.7	26.5846	53.17	79.75	106.34
.8	26.6390	53.28	79.92	106.56
.9	26.6935	53.39	80.08	106.77
98.0	26.7481	53.50	80.24	106.99
.1	26.8027	53.61	80.41	107.21
.2	26.8574	53.71	80.57	107.43
.3	26.9121	53.82	80.74	107.65
.4	26.9669	53.93	80.90	107.87
.5	27.0217	54.04	81.07	108.09
.6	27.0766	54.15	81.23	108.31
.7	27.1316	54.26	81.39	108.53
.8	27.1866	54.37	81.56	108.75
.9	27.2416	54.48	81.72	108.97
99.0	27.2968	54.59	81.89	109.19
.1	27.3519	54.70	82.06	109.41
.2	27.4072	54.81	82.22	109.63
.3	27.4625	54.92	82.39	109.85
.4	27.5178	55.04	82.55	110.07
.5	27.5732	55.15	82.72	110.29
.6	27.6286	55.26	82.88	110.51
.7	27.6841	55.37	83.05	110.74
.8	27.7397	55.48	83.22	110.96
.9	27.7953	55.59	83.39	111.18

The Young GAUGER's Instructor. 205

in ALE GALLONS.				
Depth.				
5	6	7	8	9
128.34	154.00	179.67	205.34	231.01
128.61	154.33	180.05	205.77	231.49
128.87	154.65	180.42	206.20	231.97
129.14	154.97	180.80	206.63	232.45
129.41	155.29	181.17	207.05	232.94

129.68	155.61	181.55	207.48	233.42
129.95	155.93	181.92	207.91	233.90
130.22	156.26	182.30	208.34	234.39
130.49	156.58	182.68	208.78	234.87
130.76	156.91	183.06	209.21	235.36

131.03	157.23	183.44	209.64	235.85
131.30	157.55	183.81	210.07	236.33
131.57	157.88	184.19	210.51	236.82
131.84	158.20	184.57	210.94	237.31
132.11	158.53	184.95	211.37	237.79

132.38	158.86	185.33	211.81	238.28
132.65	159.18	185.71	212.24	238.77
132.92	159.51	186.09	212.68	239.26
133.20	159.83	186.47	213.11	239.75
133.47	160.16	186.85	213.55	240.24

133.74	160.49	187.24	213.98	240.73
134.01	160.82	187.62	214.42	241.22
134.29	161.14	188.00	214.86	241.71
134.56	161.47	188.38	215.30	242.21
134.83	161.80	188.77	215.74	242.70

135.11	162.13	189.15	216.17	243.20
135.38	162.46	189.53	216.61	243.69
135.66	162.79	189.92	217.05	244.18
135.93	163.12	190.30	217.49	244.68
136.21	163.45	190.69	217.93	245.17

136.48	163.78	191.08	218.38	245.67
136.76	164.11	191.46	218.82	246.17
137.04	164.44	191.85	219.26	246.66
137.31	164.77	192.24	219.70	247.16
137.59	165.11	192.62	220.14	247.66

137.87	165.44	193.01	220.59	248.16
138.14	165.77	193.40	221.03	248.66
138.42	166.10	193.79	221.47	249.16
138.70	166.44	194.18	221.92	249.66
138.98	166.77	194.57	222.36	250.16

Areas

206 *The Young GAUGER'S Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
100 .0	27.8510	55.70	83.55	111.40
.1	27.9067	55.81	83.72	111.63
.2	27.9625	55.92	83.89	111.85
.3	28.0184	56.04	84.06	112.07
.4	28.0743	56.15	84.22	112.30
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.5	28.1302	56.26	84.39	112.52
.6	28.1862	56.37	84.56	112.74
.7	28.2423	56.48	84.73	112.97
.8	28.2984	56.60	84.90	113.19
.9	28.3546	56.71	85.06	113.42
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101 .0	28.4108	56.82	85.23	113.64
.1	28.4671	56.93	85.40	113.87
.2	28.5234	57.05	85.57	114.09
.3	28.5798	57.16	85.74	114.32
.4	28.6363	57.27	85.91	114.55
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.5	28.6928	57.39	86.08	114.77
.6	28.7494	57.50	86.25	115.00
.7	28.8060	57.61	86.42	115.22
.8	28.8627	57.73	86.59	115.45
.9	28.9194	57.84	86.76	115.68
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102 .0	28.9762	57.95	86.93	115.90
.1	29.0330	58.07	87.10	116.13
.2	29.0899	58.18	87.27	116.36
.3	29.1469	58.29	87.44	116.59
.4	29.2039	58.41	87.61	116.82
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.5	29.2610	58.52	87.78	117.04
.6	29.3181	58.64	87.95	117.27
.7	29.3753	58.75	88.13	117.50
.8	29.4325	58.86	88.30	117.73
.9	29.4898	58.98	88.47	117.96
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103 .0	29.5471	59.09	88.64	118.19
.1	29.6045	59.21	88.81	118.42
.2	29.6620	59.32	88.99	118.65
.3	29.7195	59.44	89.16	118.88
.4	29.7771	59.55	89.33	119.11
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.5	29.8347	59.67	89.50	119.34
.6	29.8924	59.78	89.68	119.57
.7	29.9501	59.90	89.85	119.80
.8	30.0079	60.02	90.02	120.03
.9	30.0657	60.13	90.20	120.26

The Young GAUGER'S Instructor. . 207

in ALE GALLONS.				
Depth.				
5	6	7	8	9
139.26	107.11	194.96	222.81	250.66
139.54	167.44	195.35	223.25	251.16
139.81	167.77	195.74	223.70	251.66
140.09	168.11	196.13	224.15	252.16
140.37	168.45	196.52	224.59	252.67
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140.65	168.78	196.91	225.04	253.17
140.93	169.12	197.30	225.49	253.67
141.21	169.45	197.70	225.94	254.18
141.49	169.79	198.09	226.39	254.69
141.77	170.13	198.48	226.84	255.19
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142.05	170.46	198.88	227.29	255.70
142.34	170.80	199.27	227.74	256.20
142.62	171.14	199.66	228.19	256.71
142.90	171.48	200.06	228.64	257.22
143.18	171.82	200.45	229.09	257.73
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143.46	172.16	200.85	229.54	258.24
143.75	172.50	201.25	229.99	258.74
144.03	172.84	201.64	230.45	259.25
144.31	173.18	202.04	230.90	259.76
144.60	173.52	202.44	231.36	260.27
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144.88	173.86	202.83	231.81	260.79
145.17	174.20	203.23	232.26	261.30
145.45	174.54	203.63	232.72	261.81
145.73	174.88	204.03	233.18	262.32
146.02	175.22	204.43	233.63	262.84
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146.31	175.57	204.83	234.09	263.35
146.59	175.91	205.23	234.54	263.86
146.88	176.25	205.63	235.00	264.38
147.16	176.59	206.03	235.46	264.89
147.45	176.94	206.43	235.92	265.41
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147.74	177.28	206.83	236.38	265.92
148.02	177.63	207.23	236.84	266.44
148.31	177.97	207.63	237.30	266.95
148.60	178.31	208.03	237.76	267.48
148.89	178.66	208.44	238.22	267.99
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149.17	179.01	209.84	238.68	268.52
149.46	179.35	209.25	239.14	269.03
149.75	179.70	209.65	239.60	269.55
150.04	180.05	210.06	240.06	270.07
150.33	180.40	210.46	240.53	270.59

Areas

208 *The Young GAUGER's Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
104.0	30.1236	60.25	90.37	120.49
.1	30.1816	60.36	90.54	120.72
.2	30.2396	60.48	90.72	120.96
.3	30.2977	60.60	90.89	121.19
.4	30.3558	60.71	91.07	121.42
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.5	30.4140	60.83	91.24	121.66
.6	30.4722	60.94	91.42	121.89
.7	30.5305	61.06	91.59	122.12
.8	30.5889	61.18	91.77	122.36
.9	30.6473	61.29	91.94	122.59
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105.0	30.7057	61.41	92.12	122.82
.1	30.7642	61.53	92.29	123.06
.2	30.8228	61.65	92.47	123.29
.3	30.8814	61.76	92.64	123.52
.4	30.9401	61.88	92.82	123.76
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.5	30.9989	62.00	93.00	124.00
.6	31.0577	62.12	93.17	124.23
.7	31.1165	62.23	93.35	124.46
.8	31.1754	62.35	93.53	124.70
.9	31.2344	62.47	93.70	124.94
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106.0	31.2934	62.59	93.88	125.17
.1	31.3525	62.70	94.06	125.41
.2	31.4116	62.82	94.23	125.65
.3	31.4708	62.94	94.41	125.88
.4	31.5300	63.06	94.59	126.12
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.5	31.5893	63.18	94.77	126.36
.6	31.6487	63.30	94.95	126.59
.7	31.7081	63.42	95.12	126.83
.8	31.7675	63.53	95.30	127.07
.9	31.8270	63.95	95.48	127.31
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107.0	31.8866	63.77	95.66	127.54
.1	31.9462	63.89	95.84	127.78
.2	32.0059	64.01	96.02	128.02
.3	32.0657	64.13	96.20	128.26
.4	32.1255	64.25	96.38	128.50
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.5	32.1853	64.37	96.56	128.74
.6	32.2452	64.49	96.74	128.99
.7	32.3052	64.61	96.92	129.22
.8	32.3652	64.73	97.10	129.46
.9	32.4252	64.85	97.28	129.70

The Young GAUGER'S Instructor. 209

in ALE GALLONS.				
Depth.				
5	6	7	8	9
150.62	180.74	210.87	240.99	271.11
150.91	181.09	211.27	241.45	271.63
151.20	181.44	211.68	241.91	272.15
151.49	181.79	212.09	242.38	272.68
151.78	182.13	212.49	242.85	273.20
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152.07	182.48	212.90	243.31	273.73
152.36	182.83	213.31	243.78	274.25
152.65	183.18	213.71	244.24	274.77
152.94	183.53	214.12	244.71	275.30
153.24	183.88	214.53	245.18	275.82
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153.53	184.23	214.94	245.65	276.35
153.82	184.58	215.35	246.11	276.88
154.11	184.94	215.76	246.58	277.41
154.41	185.29	216.17	247.05	277.93
154.70	185.64	216.58	247.52	278.46
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154.99	185.99	216.99	247.99	278.99
155.29	186.35	217.40	248.46	279.52
155.58	186.70	217.81	248.93	280.05
155.88	187.05	218.23	249.40	280.58
156.17	187.40	218.64	249.87	281.11
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156.47	187.76	219.05	250.35	281.64
156.76	188.11	219.47	250.82	282.17
157.06	188.47	219.88	251.29	282.70
157.35	188.82	220.30	251.77	283.24
157.65	189.18	220.71	252.24	283.77
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157.95	189.54	221.13	252.71	284.30
158.24	189.89	221.54	253.19	284.84
158.54	190.25	221.96	253.66	285.37
158.84	190.60	222.37	254.14	285.91
159.14	190.96	222.79	254.62	286.44
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159.43	191.32	223.21	255.09	286.98
159.73	191.68	223.62	255.57	287.52
160.03	192.04	224.04	256.05	288.05
160.33	192.40	224.46	256.53	288.59
160.63	192.75	224.88	257.00	289.13
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160.93	193.11	225.30	257.48	299.67
161.23	193.47	225.72	257.96	290.21
161.53	193.83	226.14	258.44	290.75
161.83	194.19	226.56	258.92	291.29
162.13	194.55	226.98	259.40	291.83

E e

Areas

210 *The Young GAUGER's Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
108 .0	32.4854	64.97	97.46	129.94
.1	32.5456	65.09	97.64	130.18
.2	32.6058	65.21	97.82	130.42
.3	32.6661	65.33	98.00	130.66
.4	32.7265	65.45	98.18	130.90
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.5	32.7869	65.57	98.36	131.15
.6	32.8474	65.69	98.54	131.39
.7	32.9079	65.82	98.72	131.63
.8	32.9685	65.94	98.90	131.87
.9	33.0291	66.06	99.09	132.12
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109 .0	33.0898	66.18	99.27	132.36
.1	33.1505	66.30	99.45	132.60
.2	33.2113	66.42	99.63	132.84
.3	33.2722	66.54	99.82	133.09
.4	33.3331	66.67	100.00	133.33
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.5	33.3940	66.79	100.18	133.58
.6	33.4551	66.91	100.37	133.82
.7	33.5161	67.03	100.55	134.06
.8	33.5773	67.15	100.73	134.31
.9	33.6385	67.28	100.92	134.55
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110 .0	33.6997	67.40	101.10	134.80
.1	33.7610	67.52	101.28	135.04
.2	33.8224	67.64	101.47	135.29
.3	33.8838	67.77	101.65	135.54
.4	33.9452	67.89	101.84	135.78
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.5	34.0068	68.01	102.02	136.03
.6	34.0683	68.14	102.20	136.27
.7	34.1300	68.26	102.39	136.52
.8	34.1917	68.38	102.58	136.77
.9	34.2534	68.51	102.76	137.01
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111 .0	34.3152	68.63	102.95	137.26
.1	34.3771	68.75	103.13	137.51
.2	34.4390	68.88	103.32	137.76
.3	34.5010	69.00	103.50	138.00
.4	34.5630	69.13	103.69	138.25
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.5	34.6251	69.25	103.88	138.50
.6	34.6872	69.37	104.06	138.75
.7	34.7494	69.50	104.25	139.00
.8	34.8116	69.62	104.43	139.25
.9	34.8739	69.75	104.62	139.50

in ALE GALLONS.				
Depth.				
5	6	7	8	9
162.43	194.91	227.40	259.88	292.37
162.73	195.27	227.82	260.36	292.91
163.03	195.63	228.24	260.85	293.45
163.33	196.00	228.66	261.33	293.99
163.63	196.36	229.08	261.81	294.53
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163.93	196.72	229.51	262.30	295.08
164.24	197.08	229.93	262.78	295.62
164.54	197.45	230.36	263.26	296.17
164.84	197.81	230.78	263.74	296.71
165.15	198.17	231.20	264.23	297.26
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165.45	198.54	231.63	264.72	297.81
165.75	198.90	232.05	265.20	298.35
166.06	199.27	232.48	265.69	298.90
166.36	199.63	232.90	266.18	299.45
166.67	200.00	233.33	266.66	300.00
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166.97	200.36	233.76	267.15	300.55
167.28	200.73	234.19	267.64	301.10
167.58	201.10	234.61	268.13	301.64
167.89	201.47	235.04	268.62	302.20
168.19	201.83	235.47	269.11	302.75
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168.50	202.20	235.90	269.60	303.30
168.81	202.57	236.33	270.09	303.85
169.11	202.93	236.76	270.58	304.40
169.42	203.30	237.19	271.07	304.95
169.73	203.67	237.62	271.56	305.51
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170.03	204.04	238.05	272.05	306.06
170.34	204.41	238.48	272.54	306.71
170.65	204.78	238.91	273.04	307.17
170.96	205.15	239.34	273.53	307.73
171.27	205.52	239.77	274.02	308.28
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171.58	205.89	240.21	274.52	308.84
171.89	206.26	240.64	275.02	309.39
172.20	206.63	241.07	275.51	309.95
172.51	207.01	241.51	276.01	310.51
172.82	207.38	241.94	276.50	311.07
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173.13	207.75	242.38	277.00	311.63
173.44	208.12	242.81	277.50	312.18
173.75	208.49	243.24	278.00	312.74
174.06	208.87	243.68	278.49	313.30
174.37	209.24	244.12	278.99	313.87

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
112.0	34.9363	69.87	104.81	139.75
.1	34.9987	70.00	105.00	140.00
.2	35.0612	70.12	105.18	140.24
.3	35.1237	70.25	105.37	140.50
.4	35.1863	70.37	105.56	140.75

.5	35.2489	70.50	105.75	141.00
.6	35.3116	70.62	105.93	141.25
.7	35.3744	70.75	106.12	141.50
.8	35.4372	70.87	106.31	141.75
.9	35.5000	71.00	106.50	142.00

113.0	35.5629	71.13	106.69	142.25
.1	35.6259	71.25	106.88	142.50
.2	35.6889	71.38	107.07	142.76
.3	35.7520	71.50	107.26	143.01
.4	35.8152	71.63	107.45	143.26

.5	35.8784	71.76	107.64	143.51
.6	35.9416	71.88	107.82	143.77
.7	36.0049	72.01	108.01	144.02
.8	36.0683	72.14	108.20	144.27
.9	36.1317	72.26	108.40	144.53

114.0	36.1952	72.39	108.59	144.78
.1	36.2587	72.52	108.78	145.03
.2	36.3223	72.64	108.97	145.29
.3	36.3859	72.77	109.16	145.54
.4	36.4496	72.90	109.35	145.80

.5	36.5134	73.03	109.54	146.05
.6	36.5772	73.15	109.73	146.31
.7	36.6410	73.28	109.92	146.56
.8	36.7049	73.41	110.11	146.82
.9	36.7689	73.54	110.31	147.08

115.0	36.8329	73.67	110.50	147.33
.1	36.8970	73.79	110.69	147.59
.2	36.9612	73.92	110.88	147.84
.3	37.0254	74.05	111.08	148.10
.4	37.0896	74.18	111.27	148.36

.5	37.1539	74.31	111.46	148.62
.6	37.2183	74.44	111.65	148.87
.7	37.2827	74.57	111.85	149.13
.8	37.3472	74.69	112.04	149.39
.9	37.4117	74.82	112.24	149.65

in ALE GALLONS.				
Depth.				
5	6	7	8	9
174.68	209.62	244.55	279.49	314.43
174.99	209.99	244.99	279.99	314.99
175.31	210.37	245.43	280.49	315.55
175.62	210.74	245.87	280.99	316.11
175.93	211.12	246.30	281.49	316.67
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176.24	211.49	246.84	281.99	317.24
176.56	211.87	247.18	282.49	317.80
176.87	212.24	247.62	283.00	318.37
177.19	212.62	248.06	283.50	318.93
177.50	213.00	248.50	284.00	319.50
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177.81	213.38	248.94	284.50	320.07
178.13	213.76	249.38	285.01	320.63
178.44	214.13	249.82	285.51	321.20
178.76	214.51	250.26	286.02	321.77
179.08	214.89	250.71	286.52	322.34
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179.39	215.27	251.15	287.03	322.91
179.71	215.65	251.59	287.53	323.47
180.02	216.03	252.03	288.04	324.04
180.34	216.41	252.48	288.55	324.61
180.66	216.79	252.92	289.05	325.19
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180.98	217.17	253.36	289.56	325.76
181.29	217.55	253.81	290.07	326.33
181.61	217.93	254.26	290.58	326.90
181.93	218.32	254.70	291.09	327.47
182.25	218.70	255.15	291.60	328.04
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182.57	219.08	255.59	292.11	328.62
182.89	219.46	256.04	292.62	329.19
183.21	219.85	256.49	293.13	329.77
183.53	220.23	256.93	293.64	330.34
183.85	220.61	257.38	294.15	330.92
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184.17	221.00	257.83	294.66	331.50
184.49	221.38	258.28	295.18	332.07
184.81	221.77	258.73	295.69	332.65
185.13	222.15	259.18	296.20	333.23
185.45	222.54	259.63	296.71	333.81
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185.77	222.92	260.08	297.23	334.39
186.09	223.31	260.53	297.74	334.96
186.41	223.70	260.98	298.26	335.54
186.74	224.08	261.43	298.78	336.12
187.06	224.47	261.88	299.29	336.71

Areas

214 *The Young GAUGER's Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
116.0	37.4703	74.95	112.43	149.91
.1	37.5409	75.08	112.62	150.16
.2	37.6056	75.21	112.82	150.42
.3	37.6704	75.34	113.01	150.68
.4	37.7352	75.47	113.21	150.94
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.5	37.8001	75.60	113.40	151.20
.6	37.8650	75.73	113.60	151.46
.7	37.9300	75.86	113.79	151.72
.8	37.9950	75.99	113.99	151.98
.9	38.0601	76.12	114.18	152.24
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117.0	38.1252	76.25	114.38	152.50
.1	38.1904	76.38	114.57	152.76
.2	38.2557	76.51	114.77	153.02
.3	38.3210	76.64	114.96	153.28
.4	38.3864	76.77	115.16	153.54
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.5	38.4518	76.90	115.36	153.81
.6	38.5173	77.03	115.55	154.07
.7	38.5828	77.17	115.75	154.33
.8	38.6484	77.30	115.95	154.59
.9	38.7140	77.43	116.14	154.86
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118.0	38.7797	77.56	116.34	155.12
.1	38.8455	77.69	116.54	155.38
.2	38.9113	77.82	116.73	155.64
.3	38.9772	77.95	116.93	155.91
.4	39.0431	78.09	117.13	156.17
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.5	39.1091	78.22	117.33	156.44
.6	39.1751	78.35	117.53	156.70
.7	39.2412	78.48	117.72	156.96
.8	39.3073	78.61	117.92	157.23
.9	39.3735	78.75	118.12	157.49
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119.0	39.4396	78.88	118.32	157.76
.1	39.5061	79.01	118.52	158.02
.2	39.5725	79.14	118.72	158.29
.3	39.6389	79.28	118.92	158.56
.4	39.7054	79.41	119.12	158.82
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.5	39.7719	79.54	119.32	159.09
.6	39.8385	79.68	119.52	159.35
.7	39.9052	79.81	119.72	159.62
.8	39.9719	79.94	119.92	159.89
.9	40.0386	80.08	120.12	160.15

The Young GAUGER'S Instructor. 215

<i>in ALE GALLONS.</i>				
Depth.				
5	6	7	8	9
187.38	224.80	262.33	299.81	337.29
187.70	225.25	262.79	300.33	337.87
188.03	225.63	263.24	300.84	338.45
188.35	226.02	263.69	301.36	339.03
188.68	226.41	264.15	301.88	339.62
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189.00	226.80	264.60	302.40	340.20
189.33	227.19	265.06	302.92	340.79
189.65	227.58	265.51	303.44	341.37
189.98	227.97	265.97	303.96	341.96
190.30	228.36	266.42	304.48	342.54
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190.63	228.75	266.88	305.00	343.13
190.95	229.14	267.33	305.52	343.71
191.28	229.53	267.79	306.05	344.30
191.61	229.93	268.25	306.57	344.89
191.93	230.32	268.70	307.09	345.48
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192.26	230.71	269.16	307.61	346.07
192.59	231.10	269.62	308.14	346.66
192.91	231.50	270.08	308.66	347.25
193.24	231.89	270.54	309.19	347.84
193.57	232.28	271.00	309.71	348.43
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193.90	232.68	271.46	310.24	349.02
194.23	233.07	271.92	310.76	349.61
194.56	233.47	272.38	311.29	350.20
194.89	233.86	272.84	311.82	350.79
195.22	234.26	273.30	312.34	351.39
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195.55	234.65	273.76	312.87	351.98
195.88	235.05	274.23	313.40	352.58
196.21	235.45	274.69	313.93	353.17
196.54	235.84	275.15	314.46	353.76
196.87	236.24	275.61	314.99	354.36
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197.20	236.64	276.08	315.52	354.96
197.53	237.04	276.54	316.05	355.55
197.86	237.43	277.01	316.58	356.15
198.19	237.83	277.47	317.11	356.75
198.53	238.23	277.94	317.64	357.35
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198.86	238.63	278.40	318.18	357.95
199.19	239.03	278.87	318.71	358.55
199.53	239.43	279.34	319.24	359.15
199.86	239.83	279.80	319.78	359.75
200.19	240.23	280.27	320.31	360.35

Areas

216 *The Young GAUGER's Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
120 .0	40.1054	80.21	120.32	100.42
.1	40.1723	80.34	120.52	160.69
.2	40.2392	80.48	120.72	160.96
.3	40.3062	80.61	120.92	161.22
.4	40.3733	80.75	121.12	161.49
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.5	40.4403	80.88	121.32	161.76
.6	40.5075	81.01	121.52	162.03
.7	40.5747	81.15	121.72	162.30
.8	40.6420	81.28	121.93	162.57
.9	40.7093	81.42	122.13	162.84
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121 .0	40.7766	81.55	122.33	163.11
.1	40.8441	81.69	122.53	163.38
.2	40.9116	81.82	122.73	163.65
.3	40.9791	81.96	122.94	163.92
.4	41.0467	82.09	123.14	164.19
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.5	41.1143	82.23	123.34	164.46
.6	41.1820	82.36	123.55	164.73
.7	41.2498	82.50	123.75	165.00
.8	41.3176	82.64	123.95	165.27
.9	41.3855	82.77	124.16	165.54
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122 .0	41.4534	82.91	124.36	165.81
.1	41.5214	83.04	124.56	166.09
.2	41.5895	83.18	124.77	166.36
.3	41.6575	83.31	124.97	166.63
.4	41.7257	83.45	125.18	166.90
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.5	41.7939	83.59	125.38	167.18
.6	41.8622	83.72	125.59	167.45
.7	41.9305	83.86	125.79	167.72
.8	41.9989	84.00	126.00	168.00
.9	42.0673	84.13	126.20	168.27
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123 .0	42.1358	84.27	126.41	168.54
.1	42.2043	84.41	126.61	168.82
.2	42.2729	84.55	126.82	169.09
.3	42.3416	84.68	127.02	169.37
.4	42.4103	84.82	127.23	169.64
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.5	42.4790	84.96	127.44	169.92
.6	42.5479	85.10	127.64	170.19
.7	42.6167	85.23	127.85	170.47
.8	42.6857	85.37	128.06	170.74
.9	42.7547	85.51	128.26	171.02

The Young GAUGER's Instructor. 217

in ALE GALLONS.				
Depth.				
5	6	7	8	9
200.53	240.63	280.74	320.84	360.95
200.86	241.03	281.21	321.38	361.55
201.20	241.44	281.67	321.91	362.15
201.53	241.84	282.14	322.45	362.76
201.87	242.24	282.61	322.99	363.36
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202.20	242.64	283.08	323.52	363.96
202.54	243.04	283.55	324.06	364.57
202.87	243.45	284.02	324.60	365.17
203.21	243.85	284.49	325.14	365.78
203.55	244.26	284.97	325.67	366.38
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203.88	244.66	285.44	326.21	366.99
204.22	245.06	285.91	326.75	367.60
204.56	245.47	286.38	327.29	368.20
204.90	245.87	286.85	327.83	368.81
205.24	246.28	287.33	328.37	369.42
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205.57	246.68	287.80	328.91	370.03
205.91	247.09	288.27	329.46	370.64
206.25	247.50	288.75	330.00	371.25
206.59	247.90	289.22	330.54	371.86
206.93	248.31	289.70	331.08	372.47
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207.27	248.72	290.17	331.63	373.08
207.61	249.13	290.65	332.17	373.69
207.95	249.54	291.13	332.72	374.30
208.29	249.94	291.60	333.26	374.91
208.63	250.35	292.08	333.81	375.53
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208.97	250.76	292.56	334.35	376.15
209.31	251.17	293.04	334.90	376.76
209.65	251.58	293.51	335.44	377.37
209.99	251.99	293.99	335.99	377.99
210.34	252.40	294.47	336.54	378.61
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210.68	252.81	294.95	337.09	379.22
211.02	253.23	295.43	337.63	379.84
211.36	253.64	295.91	338.18	380.46
211.71	254.05	296.39	338.73	381.07
212.05	254.46	296.87	339.28	381.69
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212.40	254.87	297.35	339.83	382.31
212.74	255.29	297.84	340.38	382.93
213.08	255.70	298.32	340.93	383.55
213.43	256.11	298.80	341.49	384.17
213.77	256.53	299.28	342.04	384.79

F f

Areas

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
124 .0	42.8237	85.65	128.47	171.29
.1	42.8928	85.79	128.68	171.57
.2	42.9619	85.92	128.89	171.85
.3	43.0312	86.06	129.09	172.12
.4	43.1004	86.20	129.30	172.40
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.5	43.1697	86.34	129.51	172.68
.6	43.2391	86.48	129.72	172.96
.7	43.3086	86.62	129.93	173.23
.8	43.3780	86.76	130.13	173.51
.9	43.4476	86.90	130.34	173.79
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125 .0	43.5172	87.03	130.55	174.07
.1	43.5868	87.17	130.76	174.35
.2	43.6566	87.31	130.97	174.63
.3	43.7263	87.45	131.18	174.91
.4	43.7961	87.59	131.39	175.18
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.5	43.8660	87.73	131.60	175.46
.6	43.9360	87.87	131.81	175.74
.7	44.0059	88.01	132.02	176.02
.8	44.0760	88.15	132.23	176.30
.9	44.1461	88.29	132.44	176.58
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126 .0	44.2162	88.43	132.65	176.86
.1	44.2865	88.57	132.86	177.15
.2	44.3567	88.71	133.07	177.43
.3	44.4271	88.85	133.28	177.71
.4	44.4974	88.99	133.49	177.99
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.5	44.5679	89.14	133.70	178.27
.6	44.6384	89.28	133.92	178.55
.7	44.7089	89.42	134.13	178.84
.8	44.7795	89.56	134.34	179.12
.9	44.8502	89.70	134.55	179.40
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127 .0	44.9209	89.84	134.76	179.68
.1	44.9916	89.98	134.97	179.97
.2	45.0625	90.12	135.19	180.25
.3	45.1334	90.27	135.40	180.53
.4	45.2043	90.41	135.61	180.82
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.5	45.2753	90.55	135.83	181.10
.6	45.3463	90.69	136.04	181.38
.7	45.4174	90.83	136.25	181.67
.8	45.4886	90.98	136.47	181.95
.9	45.5598	91.12	136.68	182.24

The Young GAUGER's Instructor. 219

in ALE GALLONS.				
Depth.				
5	6	7	8	9
214.12	250.91	299.77	342.59	385.41
214.46	257.36	300.25	343.14	386.04
214.81	257.77	300.73	343.70	386.66
215.16	258.19	301.22	344.25	387.28
215.50	258.60	301.70	344.80	387.90
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215.85	259.02	302.19	345.36	388.53
216.20	259.43	302.67	345.91	389.15
216.54	259.85	303.16	346.47	389.77
216.89	260.27	303.64	347.02	390.40
217.24	260.69	304.13	347.58	391.03
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217.59	261.10	304.62	348.14	391.65
217.93	261.52	305.11	348.69	392.28
218.28	261.94	305.60	349.25	392.91
218.63	262.36	306.08	349.81	393.53
218.98	262.78	306.57	350.37	394.16
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219.33	263.20	307.06	350.93	394.79
219.68	263.62	307.55	351.49	395.42
220.03	264.04	308.04	352.05	396.05
220.38	264.46	308.53	352.61	396.68
220.73	264.88	309.02	353.17	397.31
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221.08	265.30	309.51	353.73	397.95
221.43	265.72	310.01	354.29	398.58
221.78	266.14	310.50	354.85	399.21
222.14	266.56	310.99	355.42	399.84
222.49	266.98	311.48	355.98	400.48
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222.84	267.41	311.98	356.54	401.11
223.19	267.83	312.47	357.11	401.75
223.54	268.25	312.96	357.67	402.38
223.90	268.68	313.46	358.24	403.02
224.25	269.10	313.95	358.80	403.65
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224.60	269.53	314.45	359.37	404.29
224.96	269.95	314.94	359.93	404.92
225.31	270.37	315.44	360.50	405.56
225.67	270.80	315.93	361.07	406.20
226.02	271.23	316.43	361.63	406.84
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226.38	271.65	316.93	362.20	407.48
226.73	272.08	317.42	362.77	408.12
227.09	272.50	317.92	363.34	408.76
227.44	272.93	318.42	363.90	409.40
227.80	273.36	318.92	364.48	410.04

220 *The Young GAUGER's Instructor.*

Diam. in Inch.	Areas of Circles and Contents of Cylinders			
	Depth.			
	1	2	3	4
128 .0	45.6311	91.26	136.89	182.52
.1	45.7024	91.40	137.11	182.81
.2	45.7738	91.55	137.32	183.10
.3	45.8452	91.69	137.54	183.38
.4	45.9167	91.83	137.75	183.67
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.5	45.9883	91.98	137.96	183.95
.6	46.0599	92.12	138.18	184.24
.7	46.1315	92.26	138.39	184.53
.8	46.2032	92.41	138.61	184.81
.9	46.2750	92.55	138.83	185.10
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129 .0	46.3468	92.69	139.04	185.39
.1	46.4187	92.84	139.26	185.67
.2	46.4907	92.98	139.47	185.96
.3	46.5627	93.13	139.69	186.25
.4	46.6347	93.27	139.90	186.54
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.5	46.7068	93.41	140.12	186.83
.6	46.7790	93.56	140.34	187.12
.7	46.8512	93.70	140.55	187.40
.8	46.9235	93.85	140.77	187.69
.9	46.9958	93.99	140.99	187.98
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130 .0	47.0682	94.14	141.20	188.27
.1	47.1406	94.28	141.42	188.56
.2	47.2131	94.43	141.64	188.85
.3	47.2857	94.57	141.86	189.14
.4	47.3583	94.72	142.07	189.43
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.5	47.4309	94.86	142.29	189.72
.6	47.5037	95.01	142.51	190.01
.7	47.5764	95.15	142.73	190.31
.8	47.6493	95.30	142.95	190.60
.9	47.7222	95.44	143.17	190.89
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131 .0	47.7951	95.59	143.39	191.18
.1	47.8681	95.74	143.60	191.47
.2	47.9412	95.88	143.82	191.76
.3	48.0143	96.03	144.04	192.06
.4	48.0874	96.17	144.26	192.35
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.5	48.1606	96.32	144.48	192.64
.6	48.2339	96.47	144.70	192.94
.7	48.3073	96.61	144.92	193.23
.8	48.3806	96.76	145.14	193.52
.9	48.4541	96.91	145.36	193.82

The Young GAUGER's Instructor. 221

in ALE GALLONS.				
Depth.				
5	6	7	8	9
228.16	273.79	319.42	365.05	410.68
228.51	274.21	319.92	365.62	411.32
228.87	274.64	320.42	366.19	411.96
229.23	275.07	320.92	366.76	412.61
229.58	275.50	321.42	367.33	413.25
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229.94	275.93	321.92	367.91	413.89
230.30	276.36	322.42	368.48	414.54
230.66	276.79	322.92	369.05	415.18
231.02	277.22	323.42	369.63	415.83
231.38	277.65	323.93	370.20	416.48
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231.73	278.08	324.43	370.77	417.12
232.09	278.51	324.93	371.35	417.77
232.45	278.94	325.43	371.93	418.42
232.81	279.38	325.93	372.50	419.06
233.17	279.81	326.44	373.08	419.71
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233.53	280.24	326.95	373.65	420.36
233.90	280.67	327.45	374.23	421.01
234.26	281.11	327.96	374.81	421.66
234.62	281.54	328.46	375.39	422.31
234.98	281.97	328.97	375.97	422.96
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235.34	282.41	329.48	376.55	423.61
235.70	282.84	329.98	377.12	424.27
236.07	283.28	330.49	377.70	424.92
236.43	283.71	331.00	378.29	425.57
236.79	284.15	331.51	378.87	426.22
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237.15	284.59	332.02	379.45	426.88
237.52	285.62	332.53	380.03	427.53
237.88	285.46	333.03	380.61	428.19
238.25	285.90	333.55	381.19	428.84
238.61	286.33	334.06	381.78	429.50
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238.98	286.77	334.57	382.36	430.16
239.34	287.21	335.08	382.94	430.81
239.71	287.65	335.59	383.53	431.47
240.07	288.09	336.10	384.11	432.13
240.44	288.52	336.61	384.70	432.79
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240.80	288.96	337.12	385.28	433.45
241.17	289.40	337.64	385.87	434.11
241.54	289.84	338.15	386.46	434.77
241.90	290.28	338.66	387.04	435.43
242.37	290.72	339.18	387.43	436.09

Areas

222 *The Young GAUGER's Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
132.0	48.5276	97.00	145.58	194.11
.1	48.6011	97.20	145.80	194.40
.2	48.6747	97.35	146.02	194.70
.3	48.7484	97.50	146.25	194.99
.4	48.8221	97.64	146.47	195.29

.5	48.8959	97.79	146.69	195.58
.6	48.9697	97.94	146.91	195.88
.7	49.0436	98.09	147.13	196.17
.8	49.1176	98.23	147.35	196.47
.9	49.1916	98.38	147.57	196.77

133.0	49.2656	98.53	147.80	197.06
.1	49.3397	98.68	148.02	197.36
.2	49.4139	98.83	148.24	197.66
.3	49.4881	98.98	148.46	197.95
.4	49.5624	99.12	148.69	198.25

.5	49.6367	99.27	148.91	198.55
.6	49.7111	99.42	149.13	198.84
.7	49.7856	99.57	149.36	199.14
.8	49.8601	99.72	149.58	199.44
.9	49.9346	99.87	149.80	199.74

134.0	50.0093	100.02	150.03	200.04
.1	50.0839	100.17	150.25	200.34
.2	50.1586	100.32	150.48	200.63
.3	50.2334	100.47	150.70	200.93
.4	50.3083	100.62	150.92	201.23

.5	50.3832	100.77	151.15	201.53
.6	50.4581	100.92	151.37	201.83
.7	50.5331	101.07	151.60	202.13
.8	50.6082	101.22	151.82	202.43
.9	50.6833	101.37	152.05	202.73

135.0	50.7584	101.52	152.28	203.03
.1	50.8340	101.67	152.50	203.34
.2	50.9090	101.82	152.73	203.64
.3	50.9842	101.97	152.95	203.94
.4	51.0600	102.12	153.18	204.24

.5	51.1351	102.27	153.41	204.54
.6	51.2110	102.42	153.63	204.84
.7	51.2861	102.57	153.86	205.14
.8	51.3620	102.72	154.09	205.45
.9	51.4374	102.87	154.31	205.75

in ALE GALLONS.				
Depth.				
5	6	7	8	9
242.64	291.17	339.69	388.22	436.75
243.01	291.61	340.21	388.81	437.41
243.37	292.05	340.72	389.40	438.07
243.74	292.49	341.24	389.99	438.73
344.11	292.93	341.75	390.58	439.40
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244.48	293.38	342.27	391.17	440.06
244.85	293.82	342.79	391.76	440.73
245.22	294.26	343.30	392.35	441.39
245.59	294.71	343.82	392.94	442.06
245.96	295.15	344.34	393.53	442.72
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246.33	295.59	344.86	394.12	443.39
246.70	296.04	345.38	394.72	444.06
247.07	296.48	345.90	395.31	444.73
247.74	296.93	346.42	395.90	445.39
247.81	297.37	346.94	396.50	446.06
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248.18	297.82	347.46	397.09	446.73
248.56	298.27	347.98	397.69	447.40
248.93	298.71	348.50	398.28	448.07
249.30	299.16	349.02	398.88	448.74
249.67	299.61	349.54	399.48	449.41
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250.65	300.05	350.07	400.07	450.08
250.42	300.50	350.59	400.67	450.76
250.79	300.95	351.11	401.27	451.43
251.17	301.40	351.63	401.87	452.10
251.54	301.85	352.16	402.47	452.77
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251.92	302.30	352.68	403.07	453.45
252.29	302.75	353.21	403.66	454.12
252.67	303.20	353.73	404.26	454.80
253.04	303.65	354.26	404.87	455.47
253.42	304.10	354.78	405.47	456.15
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253.79	304.55	355.31	406.07	456.83
254.17	305.00	355.84	406.67	457.51
254.55	305.45	356.36	407.27	458.18
254.92	305.91	356.89	407.87	458.86
255.30	306.36	357.42	408.48	459.54
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255.68	306.81	357.95	409.08	460.22
256.06	307.27	358.48	409.69	460.90
256.43	307.72	359.00	410.29	461.57
256.81	308.17	359.53	410.90	462.26
257.19	308.62	360.06	411.50	462.94

Areas

224 *The Young GAUGER'S Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
136.0	51.5132	103.03	154.54	206.05
.1	51.5890	103.18	154.77	206.36
.2	51.6650	103.33	155.00	206.66
.3	51.7410	103.48	155.22	206.96
.4	51.8170	103.63	155.45	207.27
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.5	51.8930	103.79	155.68	207.57
.6	51.9690	103.94	155.91	207.88
.7	52.0450	104.09	156.14	208.18
.8	52.1210	104.24	156.36	208.48
.9	52.1972	104.39	156.59	208.79
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137.0	52.2740	104.55	156.82	209.10
.1	52.3500	104.70	157.05	209.40
.2	52.4262	104.85	157.28	209.70
.3	52.5030	105.01	157.51	210.01
.4	52.5792	105.16	157.74	210.32
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.5	52.6560	105.31	157.97	210.62
.6	52.7324	105.46	158.20	210.93
.7	52.8090	105.62	158.43	211.24
.8	52.8860	105.77	158.66	211.54
.9	52.9630	105.93	158.89	211.85
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138.0	53.0394	106.08	159.12	212.16
.1	53.1164	106.23	159.35	212.47
.2	53.1933	106.39	159.58	212.77
.3	53.2703	106.54	159.81	213.08
.4	53.3474	106.69	160.04	213.39
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.5	53.4245	106.85	160.27	213.70
.6	53.5017	107.00	160.51	214.01
.7	53.5789	107.16	160.74	214.31
.8	53.6562	107.31	160.97	214.62
.9	53.7335	107.47	161.20	214.93
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139.0	53.8109	107.62	161.43	215.24
.1	53.8884	107.78	161.67	215.55
.2	53.9659	107.93	161.90	215.86
.3	54.0434	108.09	162.13	216.17
.4	54.1211	108.24	162.36	216.48
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.5	54.1987	108.40	162.60	216.79
.6	54.2765	108.55	162.83	217.11
.7	54.3543	108.71	163.06	217.42
.8	54.4321	108.86	163.30	217.73
.9	54.5100	109.02	163.53	218.04

in ALE GALLONS.				
Depth.				
5	6	7	8	9
257.57	309.08	360.59	412.11	463.62
257.95	309.53	361.12	412.71	464.30
258.33	309.99	361.66	413.32	464.99
258.71	310.45	362.19	413.93	465.67
259.09	310.90	362.72	414.54	466.35

259.47	311.36	363.25	415.14	467.04
259.85	311.81	363.78	415.75	467.72
260.23	312.27	364.32	416.36	468.41
260.61	312.73	364.85	416.97	469.09
260.99	313.18	365.38	417.58	469.77

261.37	313.64	365.92	418.19	470.47
261.75	314.10	366.45	418.80	471.15
262.13	314.56	366.98	419.41	471.84
262.52	315.02	367.52	420.02	472.53
262.90	315.48	368.05	420.63	473.21

263.28	315.94	368.59	421.25	473.90
263.66	316.39	369.13	421.86	474.59
264.05	316.85	369.66	422.47	475.28
264.43	317.32	370.20	423.09	475.97
264.82	317.78	370.74	423.70	476.67

265.20	318.24	371.28	424.32	477.35
265.58	318.70	371.81	424.93	478.05
265.97	319.16	372.35	425.55	478.74
266.35	319.62	372.89	426.16	479.43
266.74	320.08	373.43	426.78	480.13

267.12	320.54	373.97	427.40	480.82
267.51	321.01	374.51	428.01	481.52
267.89	321.47	375.05	428.63	482.21
268.28	321.94	375.59	429.25	482.91
268.67	322.40	376.13	429.87	483.60

269.05	322.87	376.68	430.49	484.30
269.44	323.33	377.22	431.11	485.00
269.83	323.80	377.76	431.73	485.69
270.22	324.26	378.30	432.35	486.39
270.61	324.73	378.85	432.97	487.09

270.99	325.19	379.39	433.59	487.79
271.38	325.66	379.94	434.21	488.49
271.77	326.13	380.48	434.83	489.19
272.16	326.59	381.02	435.46	489.89
272.55	327.06	381.57	436.08	490.59

G g

Areas

226 · *The Young GAUGER's Instructor.*

Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
140.0	54.5880	109.18	163.76	218.35
.1	54.6660	109.33	164.00	218.66
.2	54.7440	109.49	164.23	218.98
.3	54.8222	109.64	164.47	219.29
.4	54.9003	109.80	164.70	219.60

.5	54.9786	109.96	164.94	219.91
.6	55.0569	110.11	165.17	220.23
.7	55.1352	110.27	165.41	220.54
.8	55.2136	110.43	165.64	220.85
.9	55.2921	110.58	165.88	221.17

141.0	55.3706	110.74	166.11	221.48
.1	55.4491	110.90	166.35	221.80
.2	55.5278	111.06	166.58	222.11
.3	55.6064	111.21	166.82	222.43
.4	55.6852	111.37	167.06	222.74

.5	55.7640	111.53	167.29	223.06
.6	55.8428	111.69	167.53	223.37
.7	55.9217	111.84	167.77	223.69
.8	56.0007	112.00	168.00	224.00
.9	56.0797	112.16	168.24	224.32

142.0	56.1587	112.32	168.48	224.63
.1	56.2379	112.48	168.71	224.95
.2	56.3170	112.62	168.95	225.27
.3	56.3963	112.79	169.19	225.59
.4	56.4756	112.95	169.43	225.90

.5	56.5549	113.11	169.66	226.22
.6	56.6343	113.27	169.90	226.54
.7	56.7138	113.43	170.14	226.86
.8	56.7933	113.59	170.38	227.17
.9	56.8729	113.75	170.62	227.50

143.0	56.9525	113.90	170.86	227.81
.1	57.0322	114.06	171.10	228.13
.2	57.1119	114.22	171.34	228.45
.3	57.1917	114.38	171.58	228.77
.4	57.2716	114.54	271.81	229.09

.5	57.3515	114.70	172.05	229.40
.6	57.4314	114.86	172.29	229.72
.7	57.5114	115.02	172.53	230.04
.8	57.5915	115.18	172.77	230.36
.9	57.6716	115.34	173.01	230.68

The Young GAUGER'S Instructor. 227

in ALE GALLONS.				
Depth.				
5	6	7	8	9
272.94	327.53	382.12	436.70	491.29
273.33	328.00	382.66	437.33	491.99
273.72	328.46	383.21	437.95	492.70
274.11	328.93	383.76	438.58	493.40
274.50	329.40	384.30	439.20	494.10
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274.89	329.87	384.85	439.83	494.80
275.28	330.34	385.40	440.46	495.51
275.68	330.81	385.95	441.08	496.22
276.07	331.28	386.50	441.71	496.92
276.46	331.75	387.04	442.34	497.63
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276.85	332.22	387.59	442.96	498.34
277.25	332.69	388.14	443.59	499.04
277.64	333.17	388.69	444.22	499.75
278.03	333.64	389.24	444.85	500.46
278.43	334.11	389.80	445.48	501.17
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278.82	334.58	390.35	446.11	501.88
279.21	335.06	390.90	446.74	502.59
279.61	335.53	391.45	447.37	503.30
280.01	336.00	392.00	448.01	504.01
280.40	336.48	392.56	448.64	504.72
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280.79	336.95	393.11	449.27	505.43
281.19	337.43	393.67	449.90	506.14
281.59	337.90	394.22	450.54	506.85
281.98	338.38	394.77	451.17	507.57
282.38	338.85	395.33	451.80	508.28
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282.77	339.33	395.88	452.44	508.99
283.17	339.81	396.44	453.07	509.71
283.57	340.28	397.00	453.71	510.42
283.97	340.76	397.55	454.35	511.14
284.36	341.24	398.11	454.98	511.86
-----	-----	-----	-----	-----
284.76	341.71	398.67	455.62	512.57
285.16	342.19	399.23	456.26	513.29
285.56	342.67	399.78	456.90	514.01
285.96	343.15	400.34	457.53	514.73
286.36	343.63	400.90	458.17	515.44
-----	-----	-----	-----	-----
286.76	344.11	401.46	458.81	516.16
287.16	344.59	402.02	459.45	516.88
287.56	345.07	402.58	460.09	517.60
287.96	345.55	403.14	460.73	518.32
288.36	346.03	403.70	461.37	519.04

Diam. in Inch.	Areas of Circles and Contents of Cylinders			
	Depth.			
	1	2	3	4
144.0	57.7518	115.50	173.26	231.01
.1	57.8321	115.66	173.50	231.33
.2	57.9124	115.82	173.74	231.65
.3	57.9927	115.99	173.98	231.97
.4	58.0731	116.15	174.22	232.29
.5	58.1536	116.31	174.46	232.61
.6	58.2341	116.47	174.70	232.94
.7	58.3147	116.63	174.95	233.26
.8	58.3953	116.79	175.19	233.58
.9	58.4760	116.95	175.43	233.90
145.0	58.5567	117.11	175.67	234.23
.1	58.6375	117.27	175.91	234.55
.2	58.7184	117.44	176.16	234.87
.3	58.7993	117.60	176.40	235.20
.4	58.8802	117.76	176.64	235.52
.5	58.9613	117.92	176.88	235.85
.6	59.0423	118.08	177.13	236.17
.7	59.1235	118.25	177.37	236.49
.8	59.2047	118.41	177.61	236.82
.9	59.2859	118.57	177.86	237.14
146.0	59.3672	118.73	178.10	237.47
.1	59.4485	118.90	178.35	237.79
.2	59.5299	119.06	178.59	238.12
.3	59.6114	119.22	178.83	238.45
.4	59.6929	119.39	179.08	238.77
.5	59.7745	119.55	179.32	239.10
.6	59.8561	119.71	179.57	239.42
.7	59.9378	119.88	179.81	239.75
.8	60.0196	120.04	180.06	240.08
.9	60.1014	120.20	180.30	240.41
147.0	60.1832	120.37	180.55	240.73
.1	60.2651	120.53	180.80	241.06
.2	60.3471	120.69	181.04	241.39
.3	60.4291	120.86	181.29	241.72
.4	60.5112	121.02	181.53	242.04
.5	60.5933	121.19	181.78	242.37
.6	60.6755	121.35	182.03	242.70
.7	60.7578	121.52	182.27	243.03
.8	60.8401	121.68	182.52	243.36
.9	60.9224	121.84	182.77	243.69

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in ALE GALLONS.				
Depth.				
5	6	7	8	9
288.76	346.51	404.26	462.01	519.77
289.16	346.99	404.82	462.66	520.49
289.56	347.47	405.39	463.30	521.21
289.96	347.96	405.95	463.94	521.94
290.37	348.44	406.51	464.58	522.66

290.77	348.92	407.08	465.23	523.38
291.17	349.40	407.64	465.87	524.11
291.57	349.89	408.20	466.52	524.83
291.98	350.37	408.77	467.16	525.56
292.38	350.86	409.33	467.81	526.28

292.78	251.34	409.90	468.45	527.01
293.19	351.82	410.46	469.10	527.73
293.59	352.31	411.03	469.75	528.47
294.00	352.80	411.60	470.39	529.19
294.40	353.28	412.16	471.04	529.92

294.81	353.77	412.73	471.69	530.65
295.21	354.25	413.30	472.34	531.38
295.62	354.74	413.86	472.99	532.11
296.02	355.23	414.43	473.64	532.84
296.43	355.72	415.00	474.29	533.57

296.84	356.20	415.57	474.94	534.30
297.24	356.69	416.14	475.59	535.03
297.65	357.18	416.71	476.24	535.77
298.06	357.67	417.28	476.89	536.50
298.46	358.16	417.85	477.54	537.24

298.87	358.65	418.42	478.19	537.97
299.28	359.14	418.99	478.85	538.70
299.69	359.63	419.56	479.50	539.44
300.10	360.12	420.13	480.16	540.18
300.51	360.61	420.71	480.81	540.91

300.92	361.10	421.28	481.47	541.65
301.33	361.59	421.86	482.12	542.39
301.74	362.08	422.43	482.78	543.12
302.15	362.57	423.00	483.43	543.86
302.56	363.07	423.58	484.09	544.60

302.97	363.56	424.15	484.75	545.34
303.38	364.05	424.73	485.40	546.08
303.79	364.55	425.30	486.06	546.82
304.20	365.04	425.88	486.72	547.56
304.61	365.53	426.46	487.38	548.30

Area

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Diam. in Inch.	<i>Areas of Circles and Contents of Cylinders</i>			
	Depth.			
	1	2	3	4
148.0	61.0048	122.01	183.01	244.02
.1	61.0873	122.17	183.26	244.35
.2	61.1698	122.34	183.51	244.68
.3	61.2524	122.50	183.76	245.01
.4	61.3350	122.67	184.01	245.34

.5	61.4177	122.84	184.25	245.67
.6	61.5005	123.00	184.50	246.00
.7	61.5833	123.17	184.75	246.33
.8	61.6661	123.33	185.00	246.66
.9	61.7490	123.50	185.25	247.00

149.0	61.8320	123.66	185.50	247.33
.1	61.9150	123.83	185.75	247.66
.2	61.9981	124.00	185.99	247.99
.3	62.0812	124.16	186.24	248.32
.4	62.1644	124.33	186.49	248.66

.5	62.2477	124.50	186.74	248.99
.6	62.3310	124.66	186.99	249.32
.7	62.4143	124.83	187.24	249.66
.8	62.4978	125.00	187.49	249.99
.9	62.5812	125.16	187.74	250.32

150.0	62.6648	125.33	187.99	250.66
.1	62.7483	125.50	188.24	250.99
.2	62.8320	125.66	188.50	251.33
.3	62.9157	125.83	188.75	251.66
.4	62.9994	126.00	189.00	252.00

.5	63.0832	126.17	189.25	252.33
.6	63.1671	126.33	189.50	252.67
.7	63.2510	126.50	189.75	253.00
.8	63.3350	126.67	190.01	253.34
.9	63.4190	126.84	190.26	253.68

151.0	63.5030	127.01	190.51	254.01
.1	63.5872	127.17	190.76	254.35
.2	63.6714	127.34	191.01	254.69
.3	63.7556	127.51	191.27	255.02
.4	63.8400	127.68	191.52	255.36

.5	63.9243	127.85	191.77	255.70
.6	64.0087	128.02	192.03	256.03
.7	64.0932	128.19	192.28	256.37
.8	64.1777	128.36	192.53	256.71
.9	64.2623	128.52	192.79	257.05

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in ALE GALLONS.				
Depth.				
5	6	7	8	9
305.02	366.03	427.03	488.04	594.04
305.44	366.52	427.61	488.70	549.79
305.85	367.02	428.19	489.36	550.53
306.26	367.51	428.77	490.02	551.27
306.68	368.01	429.35	490.68	552.02
-----	-----	-----	-----	-----
307.09	368.51	429.92	491.34	552.76
307.50	369.00	430.50	492.00	553.50
307.92	369.50	431.08	492.67	554.25
308.33	370.00	431.66	493.33	554.99
308.75	370.49	432.24	493.99	555.74
-----	-----	-----	-----	-----
309.16	370.99	432.82	494.66	556.49
309.58	371.49	433.41	495.32	557.24
309.99	371.99	433.99	495.98	557.98
310.41	372.49	434.57	496.65	558.73
310.82	372.99	435.15	497.32	559.48
-----	-----	-----	-----	-----
311.24	373.49	435.73	497.98	560.23
311.66	373.99	436.32	498.65	560.98
312.07	374.49	436.90	499.31	561.73
312.49	374.99	437.48	499.98	562.48
312.91	375.49	438.07	500.65	563.23
-----	-----	-----	-----	-----
313.32	375.99	438.65	501.32	563.98
313.74	376.49	439.24	501.99	564.73
314.16	376.99	439.82	502.66	565.49
314.58	377.49	440.41	503.33	566.24
315.00	378.00	441.00	504.00	566.99
-----	-----	-----	-----	-----
315.42	378.50	441.58	504.67	567.75
315.84	379.00	442.17	505.34	568.50
316.26	379.51	442.76	506.01	569.26
316.68	380.01	443.35	506.68	570.02
317.10	380.51	443.93	507.35	570.77
-----	-----	-----	-----	-----
317.52	381.02	444.52	508.02	571.53
317.94	381.52	445.11	508.70	572.28
318.36	382.03	445.70	509.37	573.04
318.78	382.53	446.29	510.04	573.80
319.20	383.04	446.88	510.72	574.56
-----	-----	-----	-----	-----
319.62	383.55	447.47	511.39	575.32
320.04	384.05	448.06	512.06	576.08
320.47	384.56	448.65	512.75	576.84
320.89	385.07	449.24	513.42	577.60
321.31	385.57	449.84	514.10	578.36

Areas

Diam. in Inch.	Areas of Circles and Contents of Cylinders			
	Depth.			
	1	2	3	4
152.0	64.3470	128.09	193.04	257.39
.1	64.4316	128.86	193.29	257.73
.2	64.5164	129.03	193.55	258.07
.3	64.6012	129.20	193.80	258.40
.4	64.6861	129.37	194.06	258.74

.5	64.7710	129.54	194.31	259.08
.6	64.8560	129.71	194.57	259.42
.7	64.9410	129.88	194.82	259.76
.8	65.0261	130.05	195.08	260.10
.9	65.1112	130.22	195.33	260.44

153.0	65.1964	130.39	195.59	260.79
.1	65.2817	130.56	195.85	261.13
.2	65.3670	130.73	196.10	261.47
.3	65.4523	130.90	196.35	261.81
.4	65.5377	131.08	196.61	262.15

.5	65.6232	131.25	196.87	262.49
.6	65.7088	131.42	197.12	262.84
.7	65.7943	131.59	197.38	263.18
.8	65.8800	131.76	197.64	263.52
.9	65.9657	131.93	197.90	263.86

154.0	66.0514	132.10	198.15	264.21
.1	66.1372	132.27	198.41	264.55
.2	66.2231	132.45	198.67	264.85
.3	66.3090	132.62	198.93	265.24
.4	66.3950	132.79	199.19	265.58

.5	66.4810	132.96	199.44	265.92
.6	66.5671	133.13	199.70	266.27
.7	66.6533	133.31	199.96	266.61
.8	66.7395	133.48	200.22	266.96
.9	66.8257	133.65	200.48	267.30

155.0	66.9120	133.82	200.74	267.65
.1	66.9984	134.00	201.00	267.99
.2	67.0848	134.17	201.25	268.34
.3	67.1713	134.34	201.51	268.69
.4	67.2578	134.12	201.77	269.03

.5	67.3444	134.69	202.03	269.38
.6	67.4311	134.86	202.29	269.72
.7	67.5178	135.03	202.55	270.07
.8	67.6045	135.21	202.81	270.42
.9	67.6913	135.38	203.07	270.77

in ALE GALLONS.				
Depth.				
5	6	7	8	9
321.74	386.08	450.43	514.78	579.12
322.16	386.59	451.02	515.45	579.88
322.58	387.10	451.61	516.13	580.65
323.01	387.61	452.21	516.81	581.41
323.43	388.12	452.80	517.49	582.17

323.86	388.63	453.40	518.17	582.94
324.28	389.14	453.99	518.85	583.70
324.71	389.65	454.59	519.53	584.47
325.13	390.16	455.18	520.21	585.23
325.56	390.67	455.78	520.89	586.00

325.98	391.18	456.37	521.57	586.77
326.41	391.69	456.97	522.25	587.54
326.84	392.20	457.57	522.94	588.30
327.26	392.71	458.17	523.62	589.07
327.69	393.23	458.76	524.30	589.84

328.12	393.74	459.36	524.99	590.61
328.54	394.25	459.96	525.66	591.38
328.97	394.77	460.56	526.35	592.15
329.40	395.28	461.16	527.04	592.92
329.83	395.79	461.76	527.73	593.69

330.26	396.31	462.36	528.41	594.46
330.69	396.82	462.96	529.10	595.23
331.12	397.34	463.56	529.78	596.01
331.55	397.85	464.16	530.47	596.78
331.98	398.37	464.77	531.16	597.56

332.41	398.89	465.37	531.85	598.33
332.84	399.40	465.97	532.54	599.10
333.27	399.92	466.57	533.23	599.88
333.70	400.44	467.18	533.92	600.66
334.13	400.95	467.78	534.60	601.43

334.56	401.47	468.38	535.30	602.21
334.99	401.99	468.99	535.99	602.99
335.42	402.50	469.59	536.68	603.76
335.86	403.03	470.20	537.37	604.54
336.29	403.55	470.80	538.06	605.32

336.72	404.07	471.41	538.76	606.10
337.16	404.59	472.02	539.45	606.88
337.59	405.11	472.62	540.14	607.66
338.06	405.63	473.23	540.83	608.44
338.46	406.15	473.84	541.53	609.22

*The Use of the TABLE of Areas of Circles,
and Contents of Cylinders.*

Seek the Diameter in the first Column on the Left-Hand, and the Depth at the Top of the Table ; then against the Diameter, and under the Depth, is the Content in Ale Gallons,

Let there be a Cylinder, whose Diameter is 30.6, and Depth 8 ; what's the Content in Ale Gallons ?

Against 30.6 in the first Column on the Left-Hand and under 8, is found 20.86, the Content in Ale Gallons.

N. B. The Depths at the Top of the Table 1, 2, 3, 4, 5, &c. do also represent .1, .2, .3, &c. or 10, 20, 30, 40, &c. or 100, 200, 300, &c. And in these Cases, the Decimal Point in the Content, is to be moved as the Depth requires. Thus in the Cylinder last given, if the Depth had been .8, the Content would be 2.086 ; if the Depth had been 80, the Content would be 208.6 ; if the Depth had been 800, the Content would be 2086.

The following Examples will render this more plain.

Let there be a Cylinder, whose Diameter is 25.7, and Depth 2.3 ; what's the Content in Ale Gallons ?

Against 25.7 and under $\begin{cases} 2 \text{ is } 3.68 \\ .3 \text{ is } .552 \end{cases}$

4.232 Ale Gallons.

Let there be a Cylinder, whose Diameter is 25.7 and Depth 23 ; what's the Content in Ale Gallons ?

Against 25.7 and under $\begin{cases} 20 \text{ is } 36.8 \\ 3 \text{ is } 5.52 \end{cases}$

42.32 Ale Gallons

Let

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Let there be a Cylinder, whose Diameter is 41.4, and Depth .7; what's the Content in Ale Gallons?

Against 41.4 and under .7 is 3.341, the Content in Ale Gallons.

Let there be a Cylinder, whose Diameter is 28.3, and Depth is 4.78; what's the Content in Ale Gallons?

$$\text{Against } 28.3 \text{ and under } \left\{ \begin{array}{l} 4 \\ .7 \\ .08 \end{array} \right\} \text{ is } \left\{ \begin{array}{l} 8.92 \\ 1.561 \\ .1784 \end{array} \right\}$$

10.6594 Ale Gallons.

Let there be a Cask, whose Mean Diameter is 29.1, and Length 46; what's the Content in Ale Gallons?

$$\text{Against } 29.1 \text{ and under } \left\{ \begin{array}{l} 40 \\ 6 \end{array} \right\} \text{ is } \left\{ \begin{array}{l} 94.3 \\ 14.15 \end{array} \right\}$$

108.45 Ale Gallons.

Let there be a Cone, whose Diameter at the Base is 54.9, and the third Part of its Height 21.2; what's the Content in Ale Gallons?

$$\text{Against } 54.9 \text{ and under } \left\{ \begin{array}{l} 20 \\ 1 \\ .2 \end{array} \right\} \text{ is } \left\{ \begin{array}{l} 167.9 \\ 8.3943 \\ 1.679 \end{array} \right\}$$

177.9733 Ale Gallons.

Let there be the Frustum of a Cone, whose lesser Diameter is 35, and greater Diameter 41.8, and Depth 38.6; what's the Content in Ale Gallons?

Greater Diameter 41.8

Lesser Diameter 35

2)76.8

Mean Diameter 38.4

H h 2

41.8

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$$\begin{array}{r} 41.8 \\ 35 \\ \hline 2)6.8 \\ \hline \end{array}$$

3.4 half Difference of the Diameters.

$$3) 38.6$$

12.86 One Third of the Depth.

Against 38.4 and under	{	30.	}	is	{	123.2
		8.	}			32.85
		.6	}			2.464
Against 3.4, and under	{	10.	}	is	{	.32196
		2.	}			.0644
		.8	}			.02575
		.06	}			.001932

158 928042 Ale Gallons.

In this last Proposition the Diameter 34 is used instead of 3.4 ; for which Reason the Tabular Numbers must be set two Places more to the Right-Hand, than if the Diameter were really 34 ; because, as the Areas of Circles, are in Proportion as the Squares of their Diameters ; therefore as there is taken ten times the Diameter, there will be had one hundred times the Area. But as this Exactness is seldom required, the Content found by the mean Diameter and Depth is near enough.

Let there be a conical Tun, whose lesser Diameter is 40.6, greater Diameter 50.8, and Depth 42 ; what's the Content in Ale Gallons ?

$$\begin{array}{r} \text{Greater Diameter } 50.8 \\ \text{Lesser Diameter } 40.6 \\ \hline 91.4 \\ \hline \text{Mean Diameter } 45.7 \end{array}$$

Against

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Against 45.7 and under $\left\{ \begin{smallmatrix} 40 \\ 2 \end{smallmatrix} \right\}$ is $\left\{ \begin{smallmatrix} 232.7 \\ 11.63 \end{smallmatrix} \right\}$

244.33 Ale Gallons.

Let there be an elliptical Tun, whose Mean Diameter is 34.6, and Depth 24; what's the Content in Ale Gallons?

Against 34.6 and under $\left\{ \begin{smallmatrix} 20 \\ 4 \end{smallmatrix} \right\}$ is $\left\{ \begin{smallmatrix} 66.7 \\ 10.0 \end{smallmatrix} \right\}$

Content 76.7 Ale Gallons

If there be a Sphere whose Diameter is 24; what's the Content in Ale Gallons?

The Content of a Sphere being two Thirds of its circumscribing Cylinder; take two Thirds of the Sphere's Diameter for the Cylinder's Depth, whose Diameter and Content is equal to that of the Sphere's.

Diameter 24
2

3)48

Depth 16

Against 24 and under $\left\{ \begin{smallmatrix} 10 \\ 6 \end{smallmatrix} \right\}$ is $\left\{ \begin{smallmatrix} 16.042 \\ 9.62 \end{smallmatrix} \right\}$

Content 25.662 Ale Gallons.

This Table may serve also for Wine Gallons, thus: Set the Diameter under itself, one Place to the Right-Hand; and again under that one Place more to the Right-Hand set half itself; add these three together, the Sum shall be the Diameter, with which you are to enter the Table for Wine Gallons.

Let

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*Let there be a Cylinder, whose Diameter is 26.8,
and Depth 36; what's the Content in Wine Gallons.*

$$\begin{array}{r} 26.8 \\ 2.68 \\ .134 \\ \hline 29.614 \end{array}$$

Against 29.6 and under $\left\{ \begin{array}{c} 30 \\ 6 \end{array} \right\}$ is $\left\{ \begin{array}{c} 73.2 \\ 14.64 \end{array} \right\}$

87.84 Wine Gallons.

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A TABLE for reducing GALLONS into BARRELS and FIRKINS, both in Beer and Ale, & contra.

Gall.	Beer	Ale	Gall.	Beer	Ale
	B. F.	B. F. G.		B. F.	B. F. G.
36	1 0	1 0 4	324	9 0	10 0 4
45	1 1	1 1 5	333	9 1	10 1 5
54	1 2	1 2 6	342	9 2	10 2 6
63	1 3	1 3 7	351	9 3	10 3 7
72	2 0	2 1 0	360	10 0	11 1 0
81	2 1	2 2 1	369	10 1	11 2 1
90	2 2	2 3 2	378	10 2	11 3 2
99	2 3	3 0 3	387	10 3	12 0 3
108	3 0	3 1 4	396	11 0	12 1 4
117	3 1	3 2 5	405	11 1	12 2 5
126	3 2	3 3 6	414	11 2	12 3 6
135	3 3	4 0 7	423	11 3	13 0 7
144	4 0	4 2 0	432	12 0	13 2 0
153	4 1	4 3 1	441	12 1	13 3 1
162	4 2	5 0 2	450	12 2	14 0 2
171	4 3	5 1 3	459	12 3	14 1 3
180	5 0	5 2 4	468	13 0	14 2 4
189	5 1	5 3 5	477	13 1	14 3 5
198	5 2	6 0 6	486	13 2	15 0 6
207	5 3	6 1 7	495	13 3	15 1 7
216	6 0	6 3 0	504	14 0	15 3 0
225	6 1	7 0 1	513	14 1	16 0 1
234	6 2	7 1 2	522	14 2	16 1 2
243	6 3	7 2 3	531	14 3	16 2 3
252	7 0	7 3 4	540	15 0	16 3 4
261	7 1	8 0 5	549	15 1	17 0 5
270	7 2	8 1 6	558	15 2	17 1 6
279	7 3	8 2 7	567	15 3	17 2 7
288	8 0	9 0 0	576	16 0	18 0 0
297	8 1	9 1 1	585	16 1	18 1 1
306	8 2	9 2 2	594	16 2	18 2 2
315	8 3	9 3 3	603	16 3	18 3 3

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A TABLE for reducing GALLONS into BARRELS and FIRKINS, both in Beer and Ale, & contra.

Gall.	Beer		Ale		Gall.	Beer		Ale	
	B. F.		B. F. G.			B. F.		B. F. G.	
612	17 0		19 0 4		900	25 0		28 0 4	
621	17 1		19 1 5		909	25 1		28 1 5	
630	17 2		19 2 6		918	25 2		28 2 6	
639	17 3		19 3 7		927	25 3		28 3 7	
648	18 0		20 1 0		936	26 0		29 1 0	
657	18 1		20 2 1		945	26 1		29 2 1	
666	18 2		20 3 2		954	26 2		29 3 2	
675	18 3		21 0 3		963	26 3		30 0 3	
684	19 0		21 1 4		972	27 0		30 1 4	
693	19 1		21 2 5		981	27 1		30 2 5	
702	19 2		21 3 6		990	27 2		30 3 6	
711	19 3		22 0 7		999	27 3		31 0 7	
720	20 0		22 2 0		1008	28 0		31 2 0	
729	20 1		22 3 1		1017	28 1		31 3 1	
738	20 2		23 0 2		1026	28 2		32 0 2	
747	20 3		23 1 3		1035	28 3		32 1 3	
756	21 0		23 2 4		1044	29 0		32 2 4	
765	21 1		23 3 5		1053	29 1		32 3 5	
774	21 2		24 0 6		1062	29 2		33 0 6	
783	21 3		24 1 7		1071	29 3		33 1 7	
792	22 0		24 3 0		1080	30 0		33 3 0	
801	22 1		25 0 1		1089	30 1		34 0 1	
810	22 2		25 1 2		1098	30 2		34 1 2	
819	22 3		25 2 3		1107	30 3		34 2 3	
828	23 0		25 3 4		1116	31 0		34 3 4	
837	23 1		26 0 5		1125	31 1		35 0 5	
846	23 2		26 1 6		1134	31 2		35 1 6	
855	23 3		26 2 7		1143	31 3		35 2 7	
864	24 0		27 0 0		1152	32 0		36 0 0	
873	24 1		27 1 1		1161	32 1		36 1 1	
882	24 2		27 2 2		1170	32 2		36 2 2	
891	24 3		27 3 3		1179	32 3		36 3 3	

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A TABLE for reducing GALLONS into BARRELS and FIRKINS, both in Beer and Ale, & contra.

Gall.	Beer	Ale	Gall.	Beer	Ale
	B. F.	B. F. G.		B. F.	B. F. G.
1188	33 0	37 0 4	1476	41 0	46 0 4
1197	33 1	37 1 5	1485	41 1	46 1 5
1206	33 2	37 2 6	1494	41 2	46 2 6
1215	33 3	37 3 7	1503	41 3	46 3 7
1224	34 0	38 1 0	1512	42 0	47 1 0
1233	34 1	38 2 1	1521	42 1	47 2 1
1242	34 2	38 3 2	1530	42 2	47 3 2
1251	34 3	39 0 3	1539	42 3	48 0 3
1260	35 0	39 1 4	1548	43 0	48 1 4
1269	35 1	39 2 5	1557	43 1	48 2 5
1278	35 2	39 3 6	1566	43 2	48 3 6
1287	35 3	40 0 7	1575	43 3	49 0 7
1296	36 0	40 2 0	1584	44 0	49 2 0
1305	36 1	40 3 1	1593	44 1	49 3 1
1314	36 2	41 0 2	1602	44 2	50 0 2
1323	36 3	41 1 3	1611	44 3	50 1 3
1332	37 0	41 2 4	1620	45 0	50 2 4
1341	37 1	41 3 5	1629	45 1	50 3 5
1350	37 2	42 0 6	1638	45 2	51 0 6
1359	37 3	42 1 7	1647	45 3	51 1 7
1368	38 0	42 3 0	1656	46 0	51 3 0
1377	38 1	43 0 1	1665	46 1	52 0 1
1386	38 2	43 1 2	1674	46 2	52 1 2
1395	38 3	43 2 3	1683	46 3	52 2 3
1404	39 0	43 3 4	1692	47 0	52 3 4
1413	39 1	44 0 5	1701	47 1	53 0 5
1422	39 2	44 1 6	1710	47 2	53 1 6
1431	39 3	44 2 7	1719	47 3	53 2 7
1440	40 0	45 0 0	1728	48 0	54 0 0
1449	40 1	45 1 1	1737	48 1	54 1 1
1458	40 2	45 2 2	1746	48 2	54 2 2
1467	40 3	45 3 3	1755	48 3	54 3 3

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A TABLE for reducing GALLONS into BARRELS and FIRKINS, both in Beer and Ale, & contra.

Gall.	Beer	Ale	Gall.	Beer	Ale
	B. F.	B. F. G.		B. F.	B. F. G.
1764	49 0	55 0 4	2052	57 0	64 0 4
1773	49 1	55 1 5	2061	57 1	64 1 5
1782	49 2	55 2 6	2070	57 2	64 2 6
1791	49 3	55 3 7	2079	57 3	64 3 7
1800	50 0	56 1 0	2088	58 0	65 1 0
1809	50 1	56 2 1	2097	58 1	65 2 1
1818	50 2	56 3 2	2106	58 2	65 3 2
1827	50 3	57 0 3	2115	58 3	66 0 3
1836	51 0	57 1 4	2124	59 0	66 1 4
1845	51 1	57 2 5	2133	59 1	66 2 5
1854	51 2	57 3 6	2142	59 2	66 3 6
1863	51 3	58 0 7	2151	59 3	67 0 7
1872	52 0	58 2 0	2160	60 0	67 2 0
1881	52 1	58 3 1	2169	60 1	67 3 1
1890	52 2	59 0 2	2178	60 2	68 0 2
1899	52 3	59 1 3	2187	60 3	68 1 3
1908	53 0	59 2 4	2196	61 0	68 2 4
1917	53 1	59 3 5	2205	61 1	68 3 5
1926	53 2	60 0 6	2214	61 2	69 0 6
1935	53 3	60 1 7	2223	61 3	69 1 7
1944	54 0	60 3 0	2232	62 0	69 3 0
1953	54 1	61 0 1	2241	62 1	70 0 1
1962	54 2	61 1 2	2250	62 2	70 1 2
1971	54 3	61 2 3	2259	62 3	70 2 3
1980	55 0	61 3 4	2268	63 0	70 3 4
1989	55 1	62 0 5	2277	63 1	71 0 5
1998	55 2	62 1 6	2286	63 2	71 1 6
2007	55 3	62 2 7	2295	63 3	71 2 7
2016	56 0	63 0 0			
2025	56 1	63 1 1			
2034	56 2	63 2 2			
2043	56 3	63 3 3			

The Use of the foregoing TABLE.

Let it be required to know, how many Barrels of Beer, or Ale, are contained in 1719 Gallons?

Seek 1719 in the Column of Gallons, and against it in the Beer Column, is 47 Barrels and three Firkins; and in the Ale Column is 53 Barrels, 2 Firkins and 7 Gallons.

Let it be required to know, how many Barrels of Beer, or Ale, are contained in 1243 Gallons?

The nearest Number to 1243 in the Table, is 1242; against which is 34 Barrels and 2 Firkins of Beer; and 38 Barrels 3 Firkins and 2 Gallons of Ale; to which the remaining 1 Gallon is to be added.

Let it be required, to know, how many Gallons are contained in 42 Barrels, 3 Firkins and 6 Gallons of Beer?

Against 42 Barrels and 3 Firkins in the Beer Column, is 1539 in the Column of Gallons, to which add 6 Gallons, the Sum is 1545 Gallons.

Let it be required to know, how many Ale Barrels are contained in 25 Barrels, 3 Firkins and 4 Gallons of Beer?

Against 25 Barrels, 3 Firkins of Beer, is 28 Barrels, 3 Firkins, 7 Gallons of Ale; to which the remaining 4 Gallons being added, the Sum is 29 Barrels and 3 Gallons of Ale.

Let it be required to know, how many Beer Barrels are contained in 59 Barrels, 1 Firkin and 7 Gallons of Ale?

Against 59 Barrels, 1 Firkin, 3 Gallons of Ale is 52 Barrels, 3 Firkins of Beer; to which the remaining 4 Gallons being added, the Sum will be 52 Barrels, 3 Firkins and 4 Gallons of Beer.

A Short Method of Reducing Ale Gallons, to Wine Gallons.

Under the given Number of Ale Gallons, set the Double thereof, one Place to the Right-Hand, the last Figure of which is to be a Decimal ; and again the same Number one Place more to the Right-Hand, the two last Figures of which are to be esteemed Decimals: Then the Sum of these three Numbers is the Quantity of Wine Gallons.

Let it be required to reduce 245 Gallons of Ale to Wine Gallons.

$$\begin{array}{r}
 245 \\
 49.0 \\
 4.9 \\
 \hline
 298.9
 \end{array}$$

Let it be required to reduce 126.5 of Ale Gallons to Wine Gallons.

$$\begin{array}{r}
 126.5 \\
 25.30 \\
 2.53 \\
 \hline
 154.33 \text{ Wine Gallons.}
 \end{array}$$





T H E

A P P E N D I X.



HEREAS it sometimes happens, that the Gauger meets with such Vessels, as are called irregular Prismoids; and as all the Authors which I have hitherto seen, are far wide of the Truth in estimating and tabulating the Contents of those Bodies, I shall first demonstrate wherein they have erred; and afterwards shew the most certain Method of Gauging those Sorts of Vessels. And,

First, Mr. *Ward*, and many others having demonstrated in the Fruustum of a Cone or Pyramid, that the Content in Inches, is equal to one Third of the Sum of the Areas of the Top, Bottom, and Mean between them, multiplied by the Depth of the Fruustum; seem to conclude the same Theorem would take Place in any irregular Prismoid. But it is not so; for the Sides of an irregular Prismoid, if continued, would not meet in a Point, but a Line; and therefore can have no Share in that Theorem. To make this more plain, Let there be a Prismoid (FIG. XXVI.) whose Top is an Oblong, the Sides of which are 16 and 18 Inches; and the Bottom is an Oblong, whose Sides are 20 and 24 Inches: Now the Dimensions of the Top and Bottom not being alike proportional, the
Sides

Sides being produced, the lesser Sides will meet in a Line, and will be in the Form of a triangular Prism cut off obliquely at each End, (as FIG. XXVII.)

Now let $AB = r$, $CD = l$, $CE = b$, $BF = h$, and $x =$ to any Abscissa.

Then $b : b :: x : \frac{bx}{b} =$ the Breadth answering to the Abscissa x .

And $b : l - r :: x : \frac{lx - rx}{b}$, then is $r + \frac{lx - rx}{b}$ the corresponding Length.

And therefore $\frac{rbx}{b} + \frac{lbxx - rbxx}{bb} =$ the Area;

which multiplied by x , will be $\frac{rbxx}{b} + \frac{lbxxx - rbxxx}{bb}$;

and this reduced to its Fluent, will be $\frac{rbxx}{2b}$

$+ \frac{lbxxx - rbxxx}{3bb}$; then substituting b for x , the Con-

tent will be $\frac{rbh}{2} + \frac{lbh - rbh}{3}$; Which is in Words

thus: To one Sixth of the Length of the Ridge; add one Third of the Length at the Bottom, this Sum multiply by the Breadth at Bottom, and this Product by the Height; this last Product is the Content in Inches, which divided by 282 for Ale, or 231 for Wine, shews the Content in Gallons.

But for the lower Frustrum (FIG. XXVI.) let $b =$ lesser Breadth, and $l =$ lesser Length, $a =$ Difference of Breadths $e =$ Difference of Lengths, and $d =$ Depth, $x =$ any Abscissa.

Then $d : a :: x : \frac{ax}{d}$; and therefore $b + \frac{ax}{d}$ the corresponding Breadth.

And $d : e :: x : \frac{ex}{d}$; and therefore $l + \frac{ex}{d}$ the corresponding Length.

Therefore

Therefore $bl + \frac{lux + bex}{d} + \frac{aexx}{dd} =$ the Area,
 which multiplied by x will be $blx + \frac{luxx + bexx}{d}$
 $+ \frac{aexxx}{dd}$; and this reduced to its Fluent will be
 $blx + \frac{luxx + bexx}{2d} + \frac{aexxx}{3dd}$; then substituting d for
 x , the Content will be $bld + \frac{l da + bde}{2} + \frac{dae}{3}$;
 which is in Words thus: Multiply Half the greater
 Breadth by the lesser Length; and Half the greater
 Length by the lesser Breadth; and one Third of
 the Difference of Lengths by the Difference of
 Breadths: Then add these three Products together,
 and multiply the Sum by the Depth; the Pro-
 duct is the Content in Inches, which divided by
 282 for Ale, or 231 for Wine, shews the Con-
 tent in Gallons.

*Let FIG. XXVI. be supposed, whose Breadth at
 the Top is 16, and Length is 18 Inches; and the
 Breadth at Bottom is 20, and Length 24 Inches; and
 Depth 30 Inches; what's the Content in Ale Gal-
 lons?*

Half the greater Breadth	10
Lesser Length	18

	180

	24
	18

Difference of Lengths	6

One Third thereof	2
Difference of Breadths	4

	8

Half

Half the greater Length 12
 Lesser Breadth 16

72

12

192

192

180

8

380

30 Depth

282)11400(40 Ale Gallons.

1128

120

Let there be a Prismoid, whose Top is a Square of 26 Inches each Side ; and the Bottom an Oblong of 26 Breadth, and 60 Length, and Depth 40 ; what's the Content in Wine Gallons ?

13

26

—

78

26

—

338

780

1118

40

231)44720(193 Wine Gallons.

231

2162

2079

830

693

137

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In this last Example; the Method commonly used, would have given but 188 Gallons.

Let there be a Prismoid, whose Top and Bottom are Triangles; whose Base at Bottom let be 48 Inches, and Perpendicular 24; and let the Base at the Top be 30 Inches, and Perpendicular 18; and let the Depth be 36 Inches; what's the Content in Wine Gallons?

$$\begin{array}{r} \text{Half of greater Base} \quad 24 \\ \text{Lesser Perpendicular} \quad 18 \\ \hline 192 \\ 24 \\ \hline 432 \end{array}$$

$$\begin{array}{r} \text{One third of Difference of Bases} \quad 6 \\ \text{Difference of Perpendiculars} \quad 6 \\ \hline 36 \end{array}$$

$$\begin{array}{r} \text{Half greater Perpendicular} \quad 12 \\ \text{Lesser Base} \quad 30 \\ \hline 360 \end{array}$$

$$\begin{array}{r} 360 \\ 432 \\ 36 \\ \hline 828 \\ 36 \\ \hline 4968 \\ 2484 \\ \hline 231 \overline{) 29808} \quad (129 \text{ Double Content.} \\ \underline{231} \\ 670 \\ 362 \\ \hline 2088 \\ 2079 \\ \hline 9 \end{array}$$

The half 64.5 Wine Gallons is the Content ; because the Triangle is half of the circumscribing Parallelogram.

Let there be a Prismoid having an elliptical Base, whose transverse Diameter is 40 Inches, and Conjugate 30 ; and the Top a Circle, whose Diameter is 30 Inches ; and the Depth 52 Inches ; what's the Content in Ale Gallons ?

Whoever has rightly perused what has been heretofore said, concerning the Reason of dividing by 359 for Ale Gallons, or 294 for Wine Gallons, will easily perceive this Figure is to be gauged as a Prismoid, whose Top is a Square and Base Parallelogram, with no other Difference, than dividing by 359 for Ale, or 294 for Wine Gallons.

15	20
30	30
---	---
450	600
600	

1050	
52	

2100	
5250	
359) 54600	(152 Ale Gallons.
359	

1870	
1795	

750	
718	

32	

Again,

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Again, to render this more conspicuous: *Let there be a Prismoid, whose Bottom and Top are Trapezia, the Diagonal at the Bottom is 64 Inches, and half Sum of the Perpendiculars 50; the Diagonal at the Top is 40 Inches, and half Sum of the Perpendiculars 31, and Depth is 60 Inches; what's the Content in Ale Gallons?*

32	25	19
31	40	8
—	—	—
32	1000	152
96		1000
—		992
992		—
		2144
		60

282)128640(456 Ale Gallons.

1128

1584

1410

1740

1692

48

From these Examples it is plain, let the Form of the Base be what it will, (if it requires no more than two Dimensions for finding its Area), the Method is the same.

It very frequently happens, that Utensils are made in the Form of an Oval, which being described in different Manners, according to the Will or Knowledge of the Workman, require a
K k 2 different

different Method of Gauging, and often lead the unwary Gauger into great Errors.

To examine an Oval, it is first of all necessary to draw the transverse and conjugate Diameters; which is thus done (FIG. XXX.) Draw within the Oval any two Lines, parallel to each other, as *ab* and *AB*, and through the Middle of these Lines, draw the Line *CD*; this Line will divide the Oval into two equal Parts, and the Middle of this Line *F* is the Center of the Oval: Now on the Point *F*, describe a Mark at *d*, and another at *g*, and draw the Line *dg*; then a Line drawn through the Center *F*, and the Middle of the Line *dg*, will be the transverse Diameter; and a Line through *F*, drawn at Right-angles to the transverse Diameter, will be the conjugate Diameter.

Now in order to examine if the given Oval be a true Ellipsis, do thus: (FIG. IV.) Take half the transverse Diameter, and set it off from *C*, to *E* and *F*, and in those Points stick Nails, or something else convenient; then bringing a String round those Nails to meet at *C*, as the Triangle *ECF* resembles, the Point *C* being moved easily round describes a true Ellipsis; whose Method of Gauging is before treated of.

Sometimes Ovals are described on two Circles, as FIG. XXVIII. and these contain more than regular Ellipses. To examine this Kind of an Oval, divide the transverse Diameter *AB* into three equal Parts at *C* and *D*; then on the Point *C*, with the Distance *AC* describe a Circle, and do the same on the Point *D*; then on the Intersections *E* and *F*, with the Diameter *AD*, describe the Arches *GH* and *IK*. If the Oval is found to be thus drawn, the following are its Properties.

T = Transverse Diameter.

C = Conjugate Diameter.

P = Circumference.

As

$$\begin{array}{l}
 \left\{ \begin{array}{l}
 1 : .756 \text{ or } 41 : 31 :: T : C. \\
 1 : .6019 :: TT : \text{Content in Inches.} \\
 1 : 2.7925 :: T : P
 \end{array} \right. \\
 \text{As } \left\{ \begin{array}{l}
 .6019 : 1 :: 282 : 468.5 \text{ Divisor for Ale Gall.} \\
 468.5 : TT :: 1 : \text{Area in Ale Gallons.} \\
 .6019 : 1 :: 231 : 383.8 \text{ Divisor for W. Gall.} \\
 383.8 : TT :: 1 : \text{Area in Wine Gallons.}
 \end{array} \right.
 \end{array}$$

$$\sqrt{468.5} = 21.64 \text{ Gauge Point for Ale Gallons.}$$

$$\sqrt{383.8} = 19.59 \text{ Gauge Point for Wine Gallons.}$$

E X P L A N A T I O N.

The Proportion of the transverse Diameter, to the conjugate Diameter, is always as 41, is to 31.

The Square of the transverse Diameter, multiplied by .6019 gives the Content in Inches.

The transverse Diameter, multiplied by 2.7925 gives the Circumference.

The Square of the transverse Diameter, divided by 468.5, gives the Area in Ale Gallons.

The Square of the transverse Diameter, divided by 383.8 gives the Area in Wine Gallons.

Against 21.64 on D, set 1 on C; then against the transverse Diameter on D, is the Area in Ale Gallons on C.

Against 19.59 on D, set 1 on C; then against the transverse Diameter on D, is the Area in Wine Gallons on C.

Another Kind of an Oval, which is common is FIG. XXIX. and this also contains more than a regular Ellipsis.

To examine if an Oval be of this Kind, on the Point A describe an Arch from C, and on that Arch set off the Distance AC to B; then through B draw the Line ED: Now if you find this Line ED equal to the conjugate Diameter, and ID equal

equal to IC, you may conclude the Oval to be of this Kind, whose Properties are as follows.

$$\begin{array}{l}
 \left\{ \begin{array}{l} 26 : 37 :: C : T \\ 1 : 1.133 :: CC : \text{Content in Inches.} \\ 1 : 3.8654 :: C : P. \end{array} \right. \\
 \text{As } \left\{ \begin{array}{l} 1.133 : 1 :: 282 : 248.3 \text{ Divisor for Ale Gall.} \\ 1.133 : 1 :: 231 : 203.9 \text{ Divisor for Wine Gall.} \\ 248.8 : 1 :: CC : \text{Area in Ale Gallons.} \\ 203.9 : 1 :: CC : \text{Area in Wine Gallons.} \end{array} \right. \\
 \sqrt{248.8} = 15.77 \text{ Gauge Point for Ale Gallons.} \\
 \sqrt{203.9} = 14.27 \text{ Gauge Point for Wine Gallons.}
 \end{array}$$

E X P L A N A T I O N.

The Proportion of the transverse Diameter, to the conjugate Diameter is always as 37, is to 26.

The Square of the conjugate Diameter, multiplied by 1,133 gives the Content in Inches.

The conjugate Diameter, multiplied by 3.8654 gives the Circumference.

The Square of the conjugate Diameter, divided by 248.8 gives the Content in Ale Gallons.

The Square of the conjugate Diameter, divided by 203 9 gives the Content in Wine Gallons.

Against 15.77 on D, set 1 on C; then against the conjugate Diameter on D, is the Area in Ale Gallons on C.

Against 14.27 on D, set 1 on C; then against the conjugate Diameter on D, is the Area in Wine Gallons on C.

These are the most common Kinds of Ovals, but there is no need to examine a Vessel in all these Forms; for if the Diameters of the given Oval be found to be in the Proportion as 37, to 26; then Examine according to the Directions last given. If the transverse and conjugate Diameters be found

to

to be in the Proportion as 41, to 31; I examine according to the Directions before given. But if the Diameters are in neither of these Proportions, I examine if the Figure be a true Ellipsis. And, lastly, if I find the Base of the Vessel to be in none of those Forms; then the following Method is to be used FIG. XXXI.

Finding this Figure in an irregular Form, I first of all (in some Part of its Curvature which seems Circular) draw the two Lines mark'd 1, and raising Perpendiculars from the Middle of these two Lines, I find them meet at A; therefore on A the Center, I try how far that Segment of a Circle reaches, and find it to reach from B to C. In the same Manner by Means of the Lines mark'd 2, I find D the Center of the Segment BE, and by Means of the Lines mark'd 3, I find F the Center of the Segment EC. Thus is the Figure divided into three circular Segments, and a Triangle, which may be gauged by the Methods before taught in this Book. And after this Manner all Ovals composed of Circular Arches, may be divided into four circular Segments, and an Oblong.

Another considerable Error among our modern Authors, is in tabulating any strait sided Figure, whose Bottom, and Top are unlike.

Mr. *Ward* and others direct thus. Divide the Difference between the top and bottom Areas by the Depth, and the Quotient will be an Addend or fixed Number; which being added to the lesser Area, the Sum will be the Area of the next Inch; and being again added to that Area, the Sum will be the Area of the third Inch, and so on.

But either this was done more for Ease than Exactness, or else those Authors did not consider, at that Time, that the Areas were in a duplicate Ratio to their Dimensions. For let us suppose a
Frustrum

Fruustum of a square Pyramid, standing on the lesser Base, whose Side is $= s$; and let the common Difference of the Side at each Inch be $= b$; then the Area of the Base will be ss , and the Area at the next Inch will be $ss + 2sb + bb$; and the Area at the third Inch will be $ss + 4sb + 4bb$.

Thus we see that the first Difference of the Areas is $2sb + bb$; but the next Difference of the Areas is $2sb + 3bb$; and therefore there is no common Addend; but if we take the Difference of the Differences of these Areas, we shall have a common Addend, and the Work stands thus:

	s squar'd	ss	
	$s + b$ squar'd	$ss + 2sb + bb$	
	$s + 2b$ squar'd	$ss + 4sb + 4bb$	
		$ss + 6sb + 9bb$	
		$ss + 8sb + 16bb$	
1st.	$2sb + bb$		$2bb$ Com. Addend.
2d.	$2sb + 3bb$	Diff. of Diffs.	$2bb$
	$2sb + 5bb$		$2bb$, &c. to the End.
	$2sb + 7bb$		

That is, in Words, find the Area of the first Inch, beginning at the lesser Base, and the Area of the second Inch, and the Area of the third Inch. Then find the Difference of the first and second Areas, and the Difference of the second and third Areas; then subtract these Differences, the lesser from the greater, the Remainder is a common Addend. The following Example will render this plain.

Let there be a Vessel, whose Form is the Fruustum of a Cone, standing on its lesser Base, whose bottom Diameter is 25.9 Inches, top Diameter 31.9 Inches, and Depth 30 Inches; then the Difference of the Diameters (6) divided by the
Depth

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Depth (30) shews the Difference of Diameters at every Inch (.2) Then the Diameter at the Middle of the

$$\begin{matrix} 1\text{st. In.} \\ 2\text{d. In.} \\ 3\text{d. In.} \end{matrix} \left. \vphantom{\begin{matrix} 1\text{st. In.} \\ 2\text{d. In.} \\ 3\text{d. In.} \end{matrix}} \right\} \text{is } \begin{matrix} 26.0 \\ 26.2 \\ 26.4 \end{matrix} \left. \vphantom{\begin{matrix} 26.0 \\ 26.2 \\ 26.4 \end{matrix}} \right\} \text{Area in } \begin{matrix} 2.29932 \\ 2.33483 \\ 2.37061 \end{matrix} \left. \vphantom{\begin{matrix} 2.29932 \\ 2.33483 \\ 2.37061 \end{matrix}} \right\} \text{W. Gall. } \begin{matrix} 1\text{st. diff.} \\ 2\text{d. diff.} \end{matrix} \left. \vphantom{\begin{matrix} 1\text{st. diff.} \\ 2\text{d. diff.} \end{matrix}} \right\} \begin{matrix} .03551 \\ .03578 \end{matrix} \text{diff. } .00027$$

Thus we find .00027 to be the common Addend, which is to be continually added to the Difference of Areas, as the following Table of the Areas at each Inch will shew.

1	2.29932	Common	16	2.86032	Common
	+ .03551	Addend.		+ .03956	Addend.
2	2.33483	.00027.	17	2.89988	.00027.
	+ .03578			+ .03983	
3	2.37061	.00027.	18	2.93971	.00027.
	+ .03605			+ .04010	
4	2.40600	.00027.	19	2.97981	.00027.
	+ .03632			+ .04037	
5	2.44298	.00027.	20	3.02018	.00027.
	+ .03650			+ .04064	
6	2.47957	.00027.	21	3.06082	.00027.
	+ .03686			+ .04091	
7	2.51643	.00027.	22	3.10173	.00027.
	+ .03713			+ .04118	
8	2.55350	.00027.	23	3.14391	.00027.
	+ .03740			+ .04145	
9	2.59090	.00027.	24	3.18436	.00027.
	+ .03767			+ .04172	
10	2.62863	.00027.	25	3.22608	.00027.
	+ .03767			+ .04199	
11	2.66657	.00027.	26	3.26807	.00027.
	+ .03821			+ .04226	
12	2.70478	.00027.	27	3.31033	.00027.
	+ .03848			+ .04253	
13	2.74326	.00027.	28	3.35286	.00027.
	+ .03875			+ .04280	
14	2.78201	.00027.	29	3.39560	.00027.
	+ .03902			+ .04307	
15	2.82103	.00027.	30	3.43873	
	+ .03929				

L 1

Then

Then by a contiual Addition of Areas, the Table for Wet Inches stands as follows.

Inches	Wine Gallons.
1	2.29934
2	4.63415
3	7.00475
4	9.41141
5	11.85439
6	14.33396
7	16.85039
8	19.40395
9	21.99491
10	24.62354
11	27.29011
12	29.99489
13	32.73815
14	35.52016
15	38.34119
16	41.20151
17	44.10139
18	47.04110
19	50.02091
20	53.04109
21	56.10191
22	59.20364
23	62.34655
24	65.53091
25	68.75699
26	72.02506
27	75.33539
28	78.68825
29	82.08391
30	85.52264

The

The Method in every strait sided Vessel is the same, having first found the Area in the Middle of the first, second, and third Inches. But if the dry Inches were to be taken of the foregoing Vessel, the common Difference must have been subtracted as followeth.

$$\text{Area of } \left\{ \begin{array}{l} \text{1st. Inch} \\ \text{2d. Inch} \\ \text{3d. Inch} \end{array} \right\} \text{ from the Top is } \left\{ \begin{array}{l} 3.43873 \\ 3.39566 \\ 3.35286 \end{array} \right\} \begin{array}{l} \text{1st. diff.} \\ \text{2d. diff.} \end{array} \left\{ \begin{array}{l} .04307 \\ .04280 \end{array} \right\} \text{ diff. } \left\{ .00027 \right.$$

1	3.43873 —04307	
	—	.00027. Common Subtrahend.
2	3.39566 —04280	
	—	.00027.
3	3.35286 —04253	
	—	.00027.
4	3.31033 —04226	
	—	
5	3.26807	And thus to the End.

Then the Areas are to be added together as before, but the Table is to be inverted.

But yet we have a Method of tabulating and computing the Contents of strait sided Vessels, howsoever irregular their Bottoms be; which take as follows:

1. After having covered the Bottom, pour in a certain Quantity of Liquor to rise as near an Inch, as conveniently may be; and exactly observe the Depth raised by the said Liquor.

2. A-

2. Again, pour in the same Quantity as before and exactly observe the Depth raised the second Time.

3. Divide the Quantity of Liquor first poured in, by the Depth raised thereby, the Quotient is the Area of the first Inch, whose square Root is to be extracted.

4. Subtract the Area of the first Inch, from the Quantity pour'd in at both Times, and divide the Remainder by the Excess of both Depths rais'd above one Inch, the Quotient is the Area of the second Inch, whose square Root is to be extracted.

5. The Difference of these two Roots is a common Addend of Roots, if the Vessel be least at Bottom, or a common subtrahend of Roots if the Vessel be biggest at Bottom, for each Inch; which Roots being squared are the Areas at each Inch.

E X A M P L E.

Let us suppose FIG. XXXII. admit I pour in five Gallons to cover the Bottom, and at the fixed dipping Place the Dip is .5.

Again, I pour in 10 Gallons and find the Dip 1.4; then consequently the ten Gallons hath rais'd .9.

Then 10 the Quantity poured in divided by .9, the Quotient is 11.1111 Gallons the Area of the first Inch.

Again, I pour in 10 Gallons, and find the Depth at the dipping Place to be 2.27; and consequently the Depth rais'd (by the last ten Gallons) above one Inch is .77; and the Quantity of Liquor poured in at both Times is 20 Gallons, from which subtract 11.1111 Gallons (the Area of the first Inch) the Remainder is 8.8889; which divided by .77, the Product is 11.5425, the Area of the second

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second Inch. Then the square Root of 11.1111 is 3.333 and the square Root of 11.5425 is 3.397; the Difference of these two Roots is the common Addend, and the Work stands thus:

		Areas
	3.333 —	11.1111
Com. Addend .064 —	3.397 —	11.5425
	.064 —	3.461 — 11.9785
	.064 —	3.525 — 12.4256
	.064 —	3.589 — 12.8809, &c.





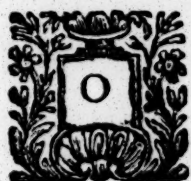
A

SUPPLEMENT

TO THE

Foregoing APPENDIX,

By the EDITORS.



F all the various Methods, which have hitherto been proposed, for approximating the Areas of curvilineal Spaces, and the Contents of Solids; none are found by Experience, to be so universally adapted to the Business of Gauging, as that of the following; which consists, principally, in the determining of a parabolic Curve, which shall pass through the Extremities of any assigned Number of equidistant perpendicular Ordinates, appertaining to any other Curve proposed: For the Area of that, determined by the common Methods, will be nearly equal to the Area of this: But in order to make this more obvious, as well as to oblige those who are conversant in the Theory, it will be necessary to premise the following Lemma and Proposition; the Methods of investigating both which, are taken from the ingenious Mr. *Simpson's* mathematical Dissertations, page 114, 115, 116.

LEM-

Plate I. to the Supp

Fig. I.

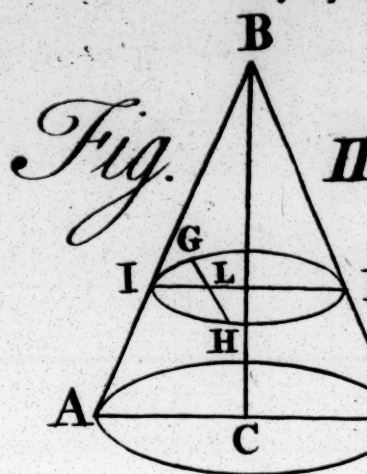
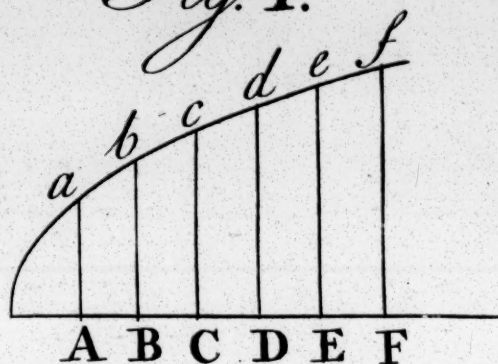


Fig. IV.

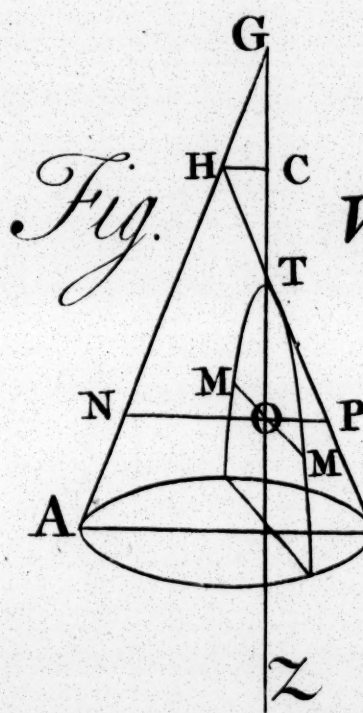
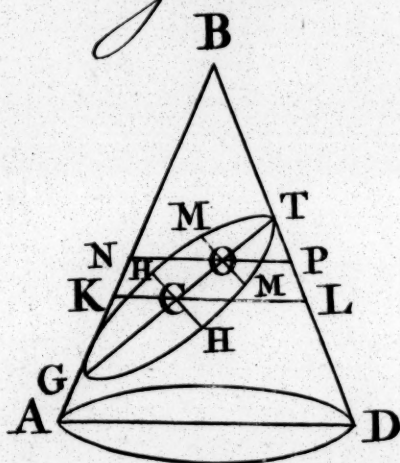


Fig. VII.

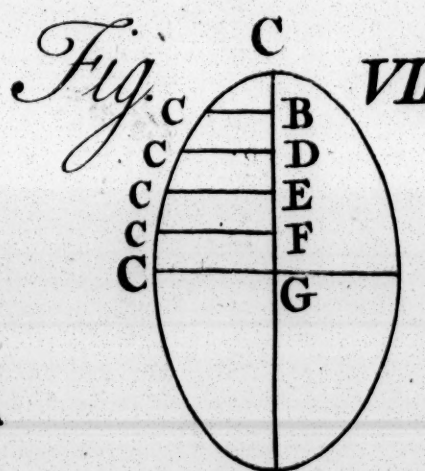
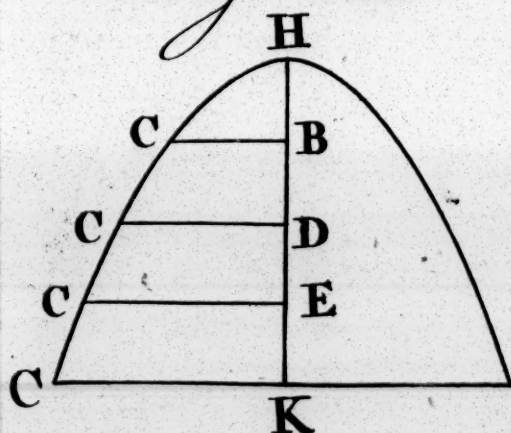
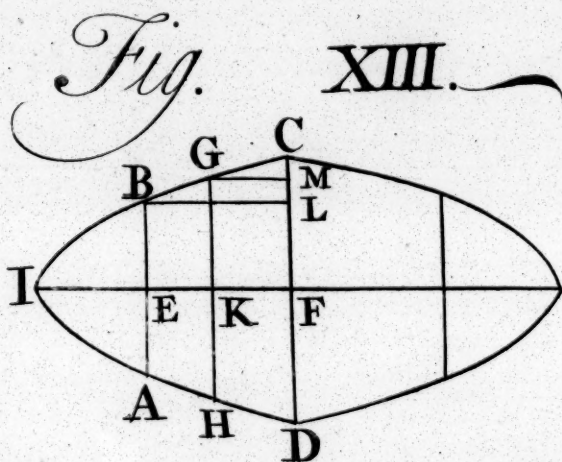
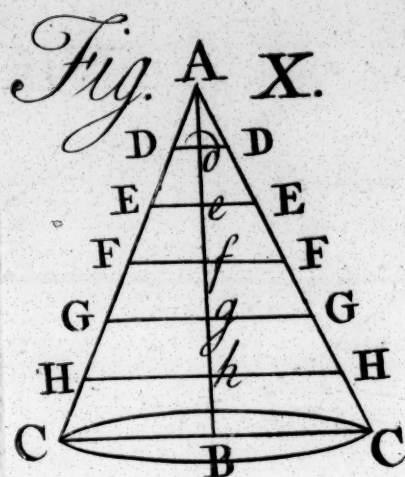
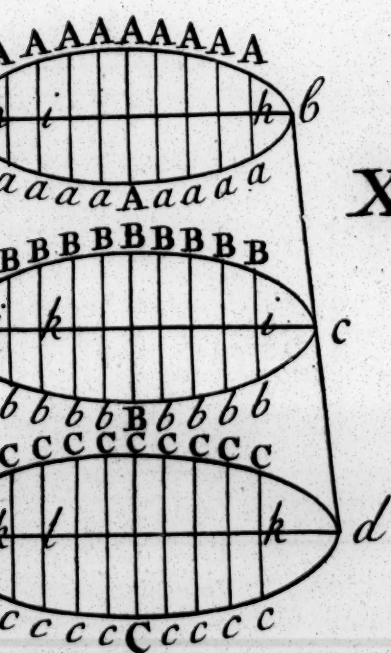
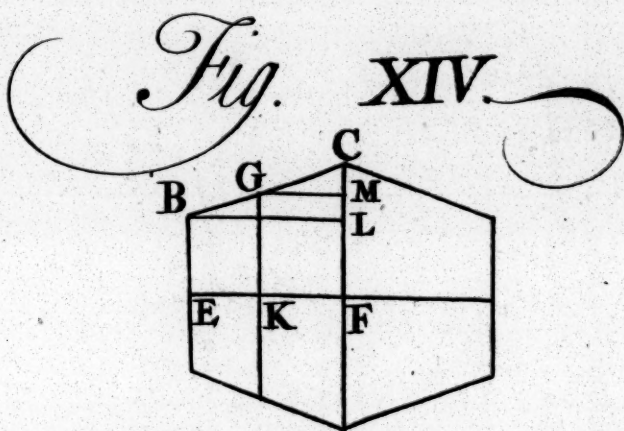
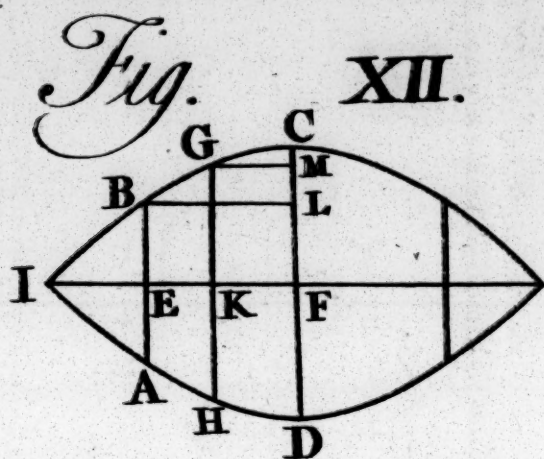
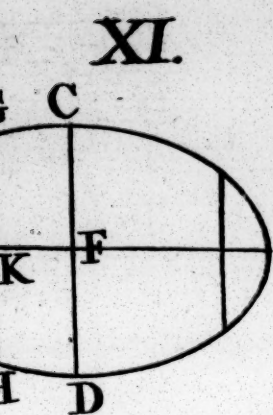
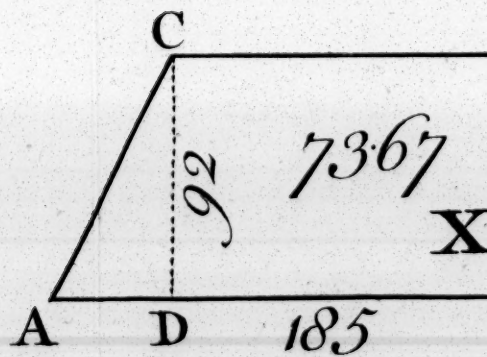
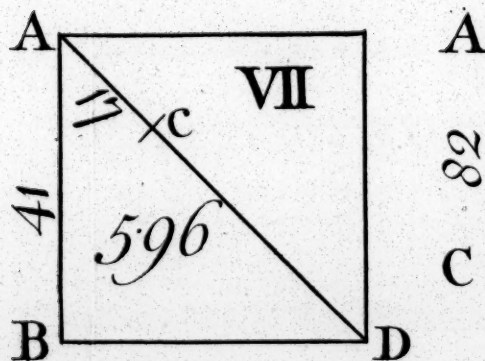
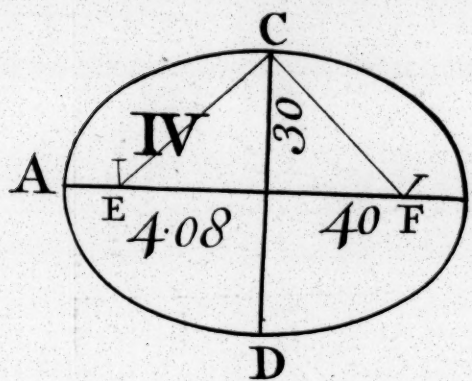
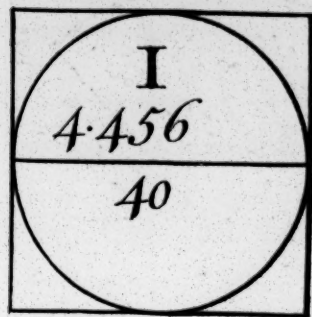


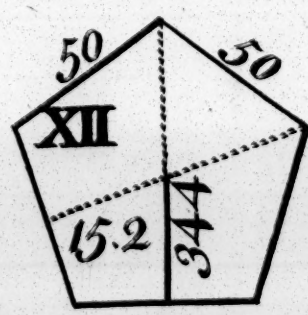
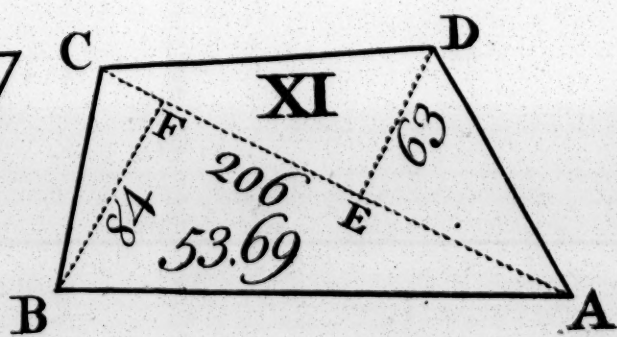
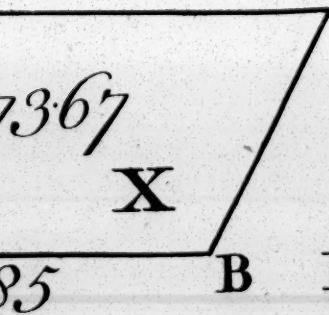
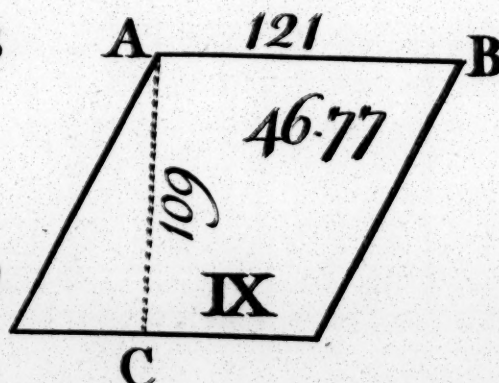
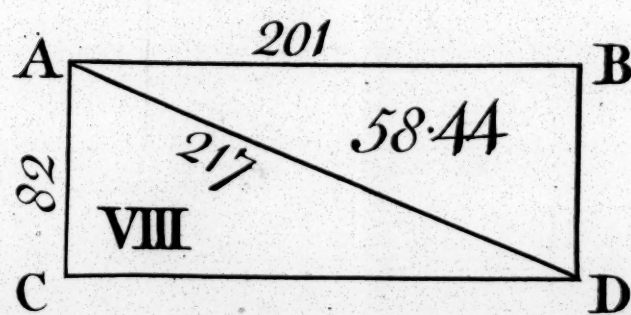
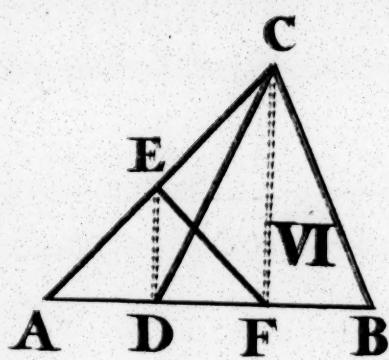
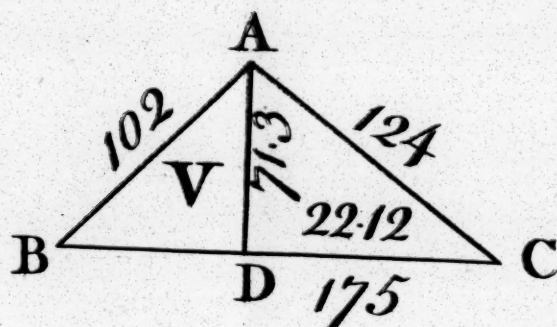
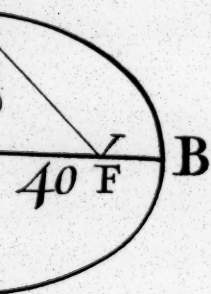
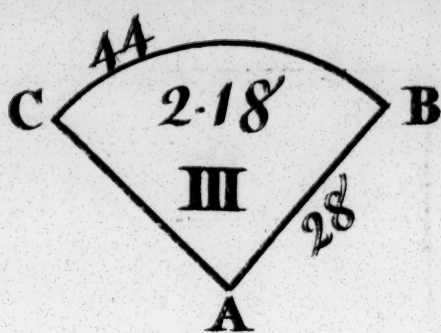
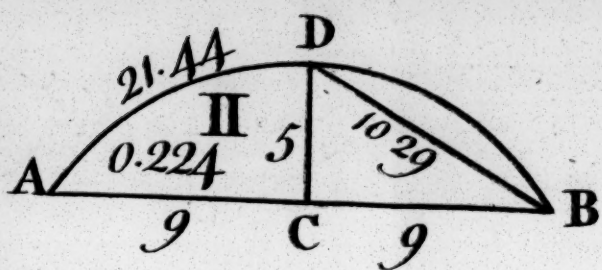
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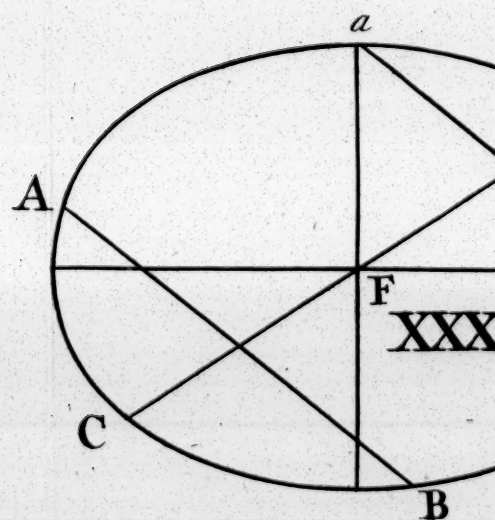
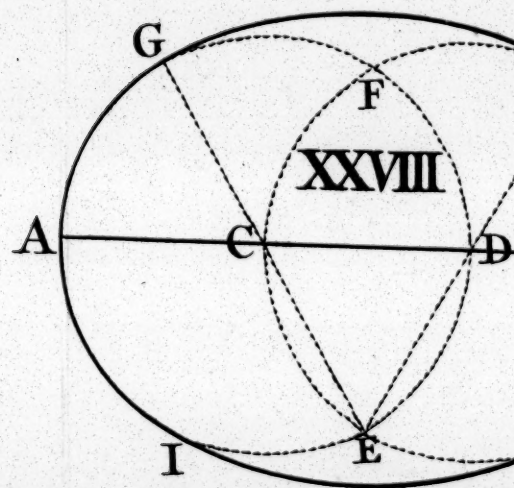
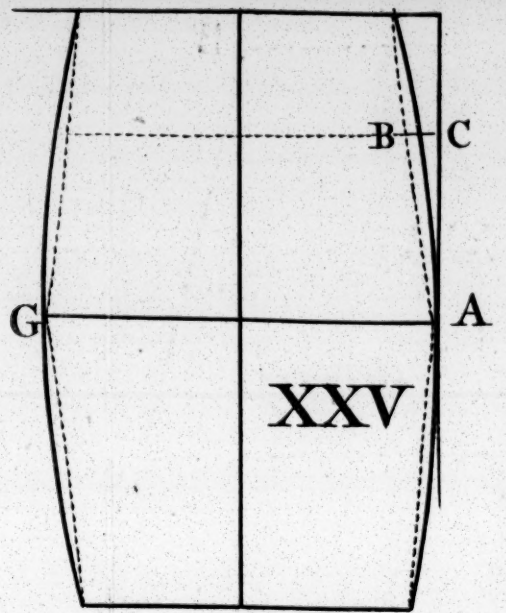


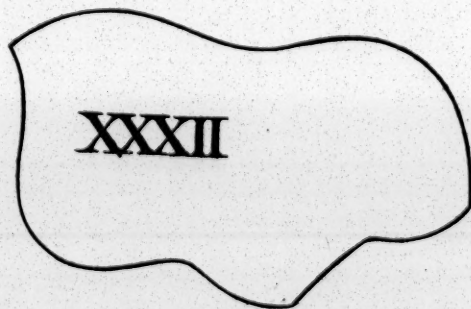
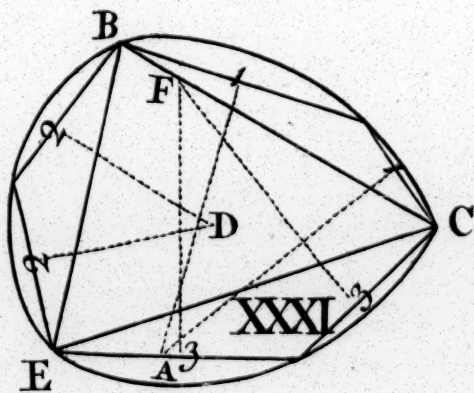
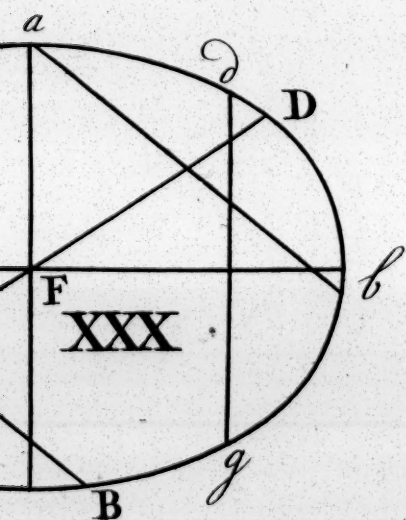
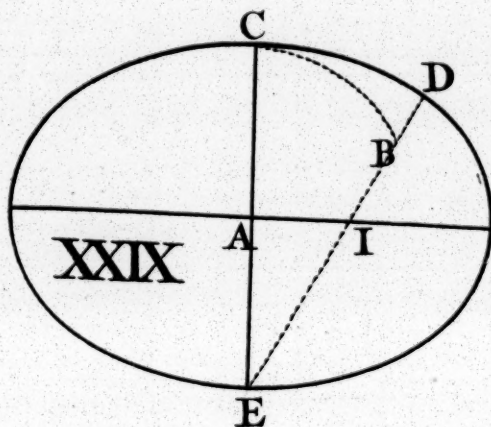
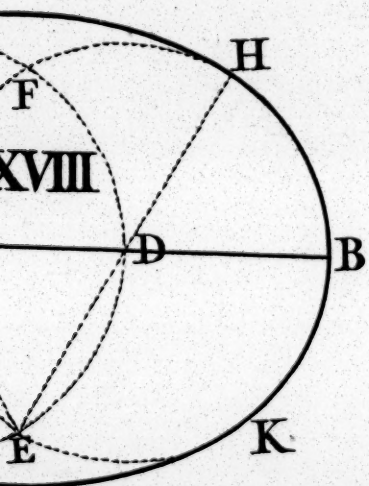
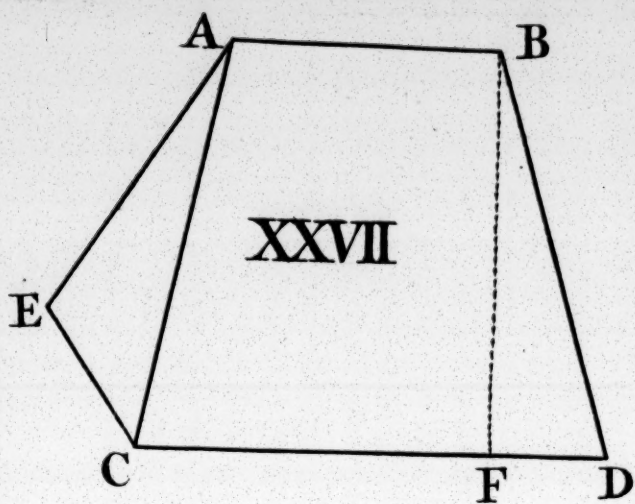
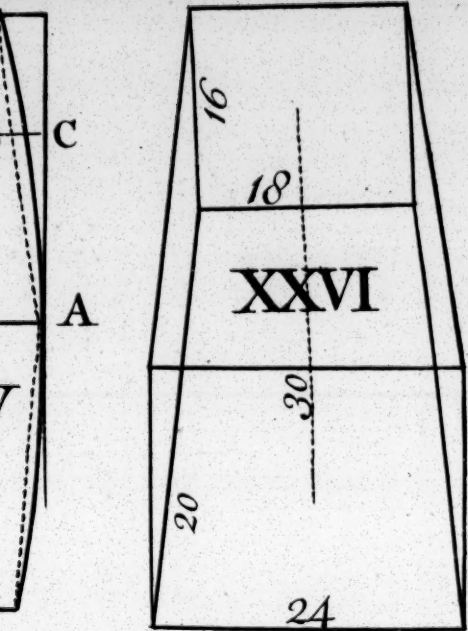
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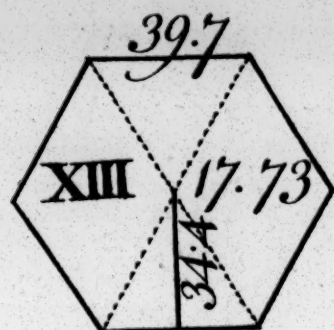




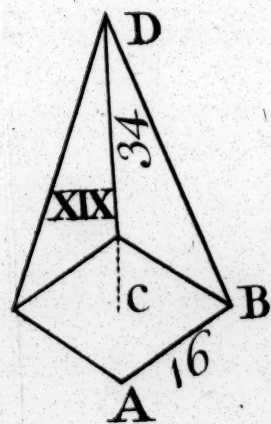


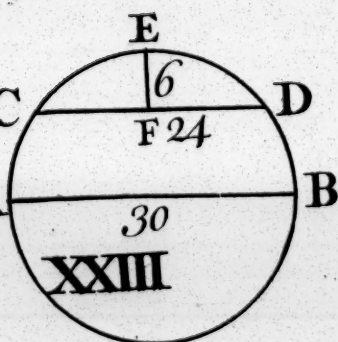
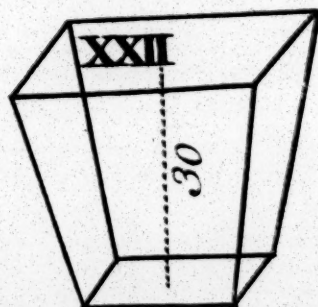
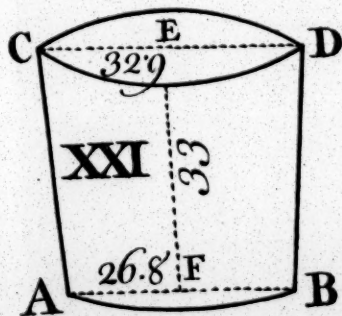
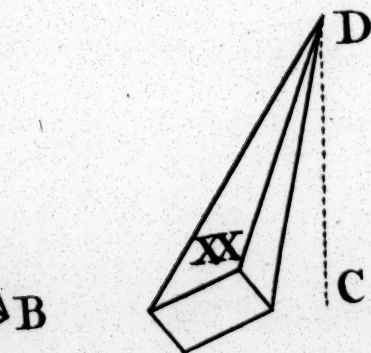
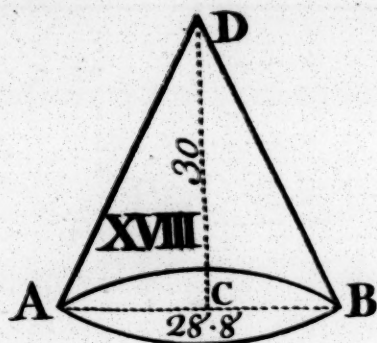
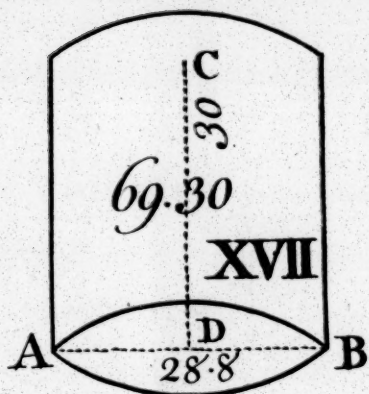
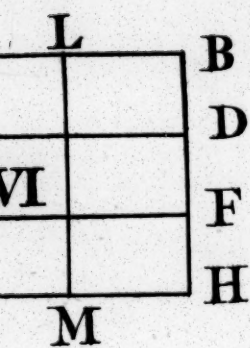
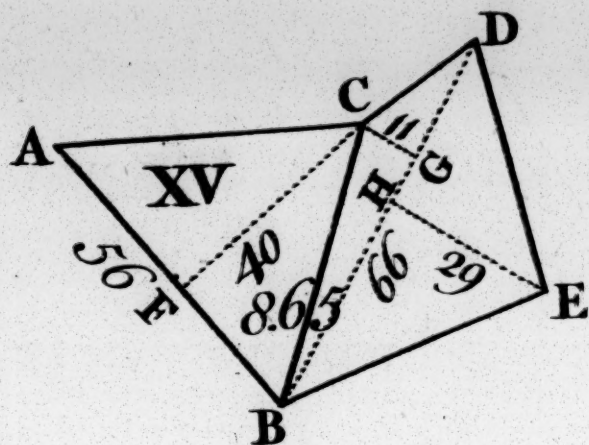
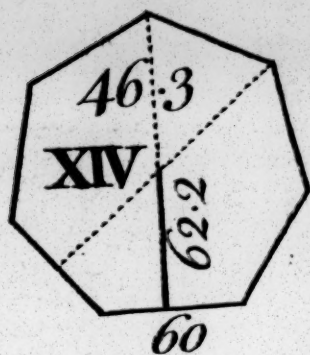
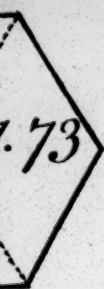






	I	L
A		
C		
E	XVI	
G		
	K	M





L E M M A.

If in any Series of Quantities $a, b, c, d, e, f, g,$
 $h, \&c.$ there be taken $A=b-a, B=c-2b+a,$
 $C=d-3c+3b-a, D=e-4d+6c-4b+a,$
 $E=f-5e+10d-10c+5b-a, F=g-6f$
 $+15e-20d+15c-6b+a, G=h-7g+$
 $21f-35e+35d-21c+7b-a, H=i-8h$
 $+28g-56f+70e-56d+28c-8b+a,$
 $\&c.$ so that the Unciæ of the Values of $A, B, C, D,$
 $\&c.$ may be those of a Binomial raised to the 1st, 2d,
 3d, 4th, $\&c.$ Powers; I say, the Value of any Term
 in that Series, whose Distance from the first is de-
 noted by n , will be $a + nA + n \cdot \frac{n-1}{2} B + n \cdot$
 $\frac{n-1}{2} \cdot \frac{n-2}{3} \cdot C + n \cdot \frac{n-1}{1} \cdot \frac{n-2}{2} \cdot \frac{n-3}{3} D, \&c.$

For since $b-a$ is $= A, C-2b+a=B,$
 $d-3c+3b-a=C, \&c.$ we shall, by Trans-
 position, have,

$$\begin{aligned}
 b &= a + A, \\
 c &= 2b - a + B, \\
 d &= 3c - 3b + a + C, \\
 e &= 4d - 6c + 4b - a + D, \\
 f &= 5e - 10d + 10c - 5b + a + E, \\
 \&c. &\qquad \&c. \qquad \&c.
 \end{aligned}$$

Where, by taking the Value of b , as found in
 the first Equation, and substituting it in the rest,
 there comes out,

$$\begin{aligned}
 c &= a + 2A + B, \\
 d &= 3c - 3A - 2a + C, \\
 e &= 4d - 6c + 4A + 3a + D, \\
 f &= 5e - 10d + 10c - 5A - 4a + E. \\
 \&c. &\qquad \&c. \qquad \&c.
 \end{aligned}$$

In

In which the Value of c , here found, being substituted, we next have,

$$\begin{aligned} d &= a + 3A + 3B + C. \\ e &= 4d - 3a - 8A - 6B + D. \\ f &= 5e - 10d + 6a + 15A + 10B + E. \\ \&c. \qquad \qquad \&c. \qquad \qquad \&c. \end{aligned}$$

In like manner the Value, of $e, f, g, h, \&c.$ are found to be $a + 4A + 6B + 4C + D$, $a + 5A + 10B + 10C + 5D + E$, $a + 6A + 15B + 20C + 15D + 6E + F$, and $a + 7A + 21B + 35C + 35D + 21E + 7F + G$, $\&c.$ Where the Unciæ, in the Value of each of the Terms $b, c, d, \&c.$ are, it is manifest, those of a Binomial raised to that Power, whose Index is equal to the Number denoting the Distance of that Term from the first in the Series; therefore the Value of that Term, whose Distance from the first, is denoted by n , will be $a + nA + n \cdot \frac{n-1}{2}$

$$B + n \cdot \frac{n-1}{2} \cdot \frac{n-2}{3} C, \&c. \text{ Q. E. D.}$$

P R O P O S I T I O N.

If $a, b, c, d, e, f, \&c.$ be a Curve of any Kind, and $Aa, Bb, Cc, Dd, Ee, Ff, \&c.$ given Ordinates thereto, at equal Distances, but not very far from each other; then the Area of the Space comprehended by that Curve, the Axis, and any two extream Ordinates, may be approximated in the following manner.

Let $Aa = a$, $Bb = b$, $Cc = c$, $Dd = d$, $\&c.$ $AB = BC = CD, \&c. = p$, and the Number of given Ordinates $Aa, Bb, Cc, \&c.$ equal to $n + 1$; putting $b - a = A$, $c - 2b + a = B$, $d - 3c + 3b - a = C$, $e - 4d + 6c - 4b + a = D$, $\&c.$ (see

(see Plate IV. Fig. 1.) as in the foregoing *Lemma*; then that Ordinate, whose Place from the first is denoted by n , or whose Distance from the first is n Times the common Distance, will (by the said *Lemma*) be $a + nA + n \cdot \frac{n-1}{2} B + n \cdot \frac{n-1}{2} \cdot \frac{n-2}{2} C + n \cdot \frac{n-1}{2} \cdot \frac{n-2}{2} \cdot \frac{n-3}{2} D$, &c.

Wherefore, if np the Distance of this Ordinate from the Point A , be put $= x$, and $\frac{x}{p}$ be substituted above instead of its equal (n) we shall have $a + \frac{Ax}{p} + \frac{Bx \cdot \frac{x-p}{2p}}{p} + \frac{Cx \cdot \frac{x-p}{2p} \cdot \frac{x-2p}{3p}}{p} + \frac{Dx \cdot \frac{x-p}{2p} \cdot \frac{x-2p}{3p} \cdot \frac{x-3p}{4p}}{p} + \frac{Ex \cdot \frac{x-p}{2p} \cdot \frac{x-2p}{3p} \cdot \frac{x-3p}{4p} \cdot \frac{x-4p}{5p}}{p} + \frac{Fx \cdot \frac{x-p}{2p} \cdot \frac{x-2p}{3p} \cdot \frac{x-3p}{4p} \cdot \frac{x-4p}{5p} \cdot \frac{x-5p}{6p}}{p}$, &c. equal to

(y) that Ordinate whose corresponding Absciss is x ; which reduced to simple Terms, will be $a + \frac{Ax}{p} + \frac{Bx^2}{2p^2} - \frac{Bx}{2p} + \frac{Cx^3}{p^3} - \frac{Cx^2}{2p^2} + \frac{Cx}{3p} + \frac{Dx^4}{24p^4} - \frac{Dx^3}{4p^3} + \frac{11Dx^2}{24p^2} - \frac{Dx}{4p} + \frac{Ex^5}{120p^5} - \frac{Ex^4}{12p^4} + \frac{7Ex^3}{24p^3} - \frac{5Ex^2}{12p^2} + \frac{Ex}{5p}$, &c.

Hence it is manifest, that $a + \frac{Ax}{p} + \frac{Bx^2}{2p^2} - \frac{Bx}{2p} + \frac{Cx^3}{p^3} - \frac{Cx^2}{2p^2} + \frac{Cx}{3p} + \frac{Dx^4}{24p^4}$, &c. $= y$ is the Equation of a parabolic Curve, which being described to the Absciss AG , will pass through all the given Points a, b, c, d , &c.

Therefore the Area of this Curve, which by the common Methods is found to be x into $a + \frac{Ax}{2p} + \frac{Bx^2}{3 \cdot 2p^2} - \frac{Bx}{2 \cdot p^2} + \frac{Cx^3}{4 \cdot 6p^3} - \frac{Cx^2}{3 \cdot 2p^2} + \frac{Cx}{2 \cdot 3p} + \frac{Dx^4}{5 \cdot 24p^4} - \frac{Dx^3}{4 \cdot 4p^3} + \frac{11Dx^2}{2 \cdot 24p^2} - \frac{Dx}{4 \cdot p} + \frac{Ex^5}{6 \cdot 120p^5} - \frac{Ex^4}{5 \cdot 12p^4} + \frac{7Ex^3}{3 \cdot 24p^3} - \frac{5Ex^2}{5 \cdot 12p^2} + \frac{Ex}{5 \cdot p}$, &c.

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$$\begin{aligned} & \frac{D x^4}{4.5.6.} - \frac{D x^3}{4.4 p^3} + \frac{11 D x}{3.4.6 p^2} - \frac{D x}{2.4 p} + \frac{E x^5}{4.5.6.6} - \\ & \frac{E x^4}{3.4.5 p 4.} + \frac{7 E x^3}{4.4.6 p^3} - \frac{5 E x^2}{3.3.4} + \frac{E x}{2.5 p} + \frac{F x 6}{4.5.6.6.7 p^6} \\ & - \frac{F x^5}{3.4.4.6 p^5} + \frac{17 F x}{2.3.4.5.6 p^4} - \frac{5 F x^3}{2.4.8 p^3} + \frac{137 F x^2}{3.6.6.10 p^2} \\ & + \frac{E x}{2.6 p} + \frac{G x^7}{4.5.6.6.8 p^7}, \&c. \text{ must be equal very} \\ & \text{nearly to the Area of the proposed Space } A a b c d \\ & G A. Q. E. I. \end{aligned}$$

COROLLARY I.

Hence, if x be taken equal to $p, 2 p, 3 p, 4 p, 5 p, 6 p, 7 p, \&c.$ $A, B, C, D, E, F, \&c.$ as in the *Prop.* or *Lemma*; the Number of Ordinates, and Areas corresponding thereto, will come out as in the following Table.

Ordinates.	Areas.
2	$\frac{b + a \times x,}{2}$
3	$\frac{a + 4 b + c \times x,}{6}$
4	$\frac{a + 3 b + 3 c + d \times x,}{8}$
5	$\frac{7 a + 32 b + 12 C + 32 d + 7 e \times x,}{90}$
6	$\frac{19 a + 75 b + 50 c + 50 d + 75 e + 19 f \times x,}{288}$
7	$\frac{41 a + 216 b + 27 C + 272 d + 27 e + 216 f + 41 g \times x,}{840}$
8	$\frac{751 a + 3577 b + 1323 C + 2989 d + 2989 e + 1323 f + 3577 g + 751 h \times x,}{17280}$
&c.	&c.

Where

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Where x in each Case denotes the Distance of the two extreme Ordinates: Now if A be put equal to the Sum of the first and last Ordinates, B the Sum of those next to them, C the Sum of the two next following the last, and so on; then the foregoing Table may be expressed thus.

Ordin. Areas.

2	$\frac{A \times x}{2},$
3	$\frac{A + 3 B \times x}{6},$
4	$\frac{A + 3 B \times x}{8},$
5	$\frac{7 A + 32 B + 12 C \times x}{90},$
6	$\frac{19 A + 75 B + 50 C \times x}{288},$
7	$\frac{41 A + 216 B + 27 C + 272 D \times x}{840},$
8	$\frac{751 A + 3577 B + 1323 C + 2989 D \times x}{17280},$
&c	&c.

Where x in each Case denotes the same as before: But if the Area be taken from the Vertex of the Curve; then is $a = 0$; wherefore the Table immediately preceding the last will become,

M m 2

Ordin.

Ordin.	Areas.
1	$\frac{b \times x}{2},$
2	$\frac{4b + c \times x}{6},$
3	$\frac{3b + 3c + d \times x}{8},$
4	$\frac{3b + 12c + 32d + 7e \times x}{90},$
5	$\frac{75b + 50c + 50d + 75e + 19f \times x}{288},$
6	$\frac{216b + 27c + 272d + 27e + 21bf + 41g \times x}{840},$
7	$\frac{3577b + 1323c + 2989e + 1323f + 3577g + 751h \times x}{17280},$
&c.	&c.

Where x in each Case, now, denotes the Distance from the Vertex of the Curve, to the last Ordinate.

Having shewn, in general, the Method of constructing the Tables, their Uses come next in Order to be insisted on; but before we proceed to that, it may not amiss to give the Definition of a Cone, and the Properties of its Sections, as the Author has no where taken Notice of them, in the former Part of this Work.

DEFINITION.

A *Cone*, ABD , (FIG. 2.) is a Solid having a circular Base, every Section parallel thereto decreasing continually, till it terminates in a Point, B ,
called

called the Vertex : And may easily be conceived to be formed, by the Revolution of a right-angled Triangle, ABC , about one of its Legs, BC , considered as immoveable.

1. If a Cone (FIG. 2.) be cut by a Plane parallel to the Base ; the Figure of the Section $GHIK$ will be that of a Circle : Therefore, in which, if any two Lines, GH, IK , be drawn, intersecting each other ; the Rectangle comprehended under the Segments, GL, LH , of the one, will (by Eu. 35. 3.) be equal to the Rectangle contained under the Segments, IL, LK , of the other ; that is, $GL \times LH = IL \times LK$.

2. If one Side, BD , of a Cone (FIG. 3.) be cut by a Plane in a Direction parallel to the other Side, AB ; the Figure of the Section $GHGK$ will be that of a Parabola : In which, if a right Line HL be drawn from the Vertex, H , dividing the Area of the Section into two equal Parts, it is called the Axis ; any right Line MN perpendicular to the Axis terminated by the Curve, is an Ordinate, and the Distance HN in the Axis, from the Vertex, to the Intersection of that Ordinate, is called an Absciss : But in order to determine the Property of this Curve ; let $P N Q$ be parallel to the base Diameter AD , cutting the Axis HL of the Parabola in the Point N ; then will the Triangles DHK, HNQ be equiangular, and therefore $HK : HN :: DK : NQ$; whence (by Eu. xvi. 6.) $HK \times NQ = HN \times DK$; but AK being equal to NP , by Construction ; therefore will $HK \times NQ \times NP = HN \times DK \times AK$; but since the Points A, G, D, G , are placed in the Periphery of a Circle, and also the Points P, M, Q, M , in that of another Circle, parallel to the former ; it follows, from the Property of the Circle, just now mentioned, that $NQ \times NP = MN^2$ and $AK \times DK = GK^2$; consequently $HK \times MN^2 = HN \times GK^2$; or

HK

$HK : HN :: GK^2 : MN^2$; that is, the Squares of the Ordinates are one to another, as their corresponding Abscissas.

3. If one Side, BD , of a Cone (FIG. 4.) be cut by a Plane in any Direction, but that which is parallel to the Base, till it passës out at the other Side AB ; the Section $GHGH$ formed thereby, will be that of an Ellipsis. In which, if two right Lines TG , MM , be drawn bisecting each other perpendicularly, and also the Area of the Section; the longer is called the Transverse, and the other Conjugate Axis; any right Line drawn perpendicular to the Axis, terminated by the Curve, is an Ordinate, and the Distance GO in the Axis, from the Vertex, to the Intersection of that Ordinate, is called an Absciss. But in order to determine the Property of this Curve; thro' the Points O, C , draw the Lines NOP , KCL parallel to the base Diameter AD ; then will the Triangles GKC , TOP and GNO , TCL be respectively equiangular; therefore $CG : CK :: GO : NO$ and $CT : CL :: OT : OP$; whence by Composition $CG \times CT : CK \times CL :: GO \times OT : NO \times OP$; but by Construction $CG = CT$; also, by the Property of the Circle, $CK \times CL = CH^2$, and $NO \times OP = MO^2$; consequently $CT^2 : CH^2 :: GO \times OT : MO^2$; Or, $MO^2 : GO \times OT :: 4 \times CH^2 : 4 \times CT^2$; that is, the Square of any Ordinate, to the transverse Axe, is to the Rectangle of the Abscissas, which it divides; as the Square of the Conjugate, to the Square of the transverse Axe.

4. If one Side of a Cone (FIG. 5.) be cut by a Plane in a Direction parallel to its Axis; the Figure of the Section, $MTMO$, thereby produced, will be that of an Hyperbola. In which if a right Line TZ , be drawn from the Vertex, T , dividing the Area of the Section into two equal Parts; it is called the Axis; any right Line MO , perpendicular to the

the

the Axis, terminated by the Curve, is called an Ordinate, and the Distance TO in the Axis, from the Vertex T , to the Intersection of that Ordinate, is called an Absciss; moreover, the right Line GT intercepted between the Vertex T , and the Point where the Absciss, and the other Side of the Cone, both produced, intersect, is called the transverse Axe, and the right Line, CH , drawn from the Vertex of the Cone bisecting it, at right Angles, is called the semi-conjugate Axe: But in Order to determine the Property of this Curve, or the Relation between the Abscissas, and Ordinates in general; thro' the Point O , draw the Line $NO P$ parallel to the base Diameter AD ; then the Triangles CHG , CHT , and GNO , OPT will be respectively equiangular; therefore, $CG:CH::GO:NO$, and $CT:CH::OT:OP$; whence by Composition $CG \times CT:CH^2::GO \times OT:NO \times OP$; but by Construction $CG = CT$; also, by the Property of the Circle, $NO \times OP = MO^2$; wherefore $CT^2:CH^2::GO \times OT:MO^2$; or $4 \times CT^2:4 \times CH^2::GO \times OT:MO^2$; that is, *the Square of the transverse Axe, is to the Square of the conjugate Axe; as the Rectangle of the Transverse, added to the Abscissa, to the Square of the Ordinate applied to that Abscissa.*

So many Properties of the Sections of a Cone, being demonstrated as seem necessary for the present Design; we shall now return from this Digression, and proceed to shew the Uses of the Tables.

Example 1. *In which it is proposed, to find the Area of a Circle, whose Diameter is 40 Inches. Let the Semi-diameter AG (FIG. 6.) be divided into 5 equal Parts, or more, according as it is proper to use this, or that Theorem in the Tables; but we shall instance here only in 5, and each will be*

be 4; wherefore by the Property of the Circle in Page 269,

$$\text{The } \left\{ \begin{array}{l} 1\text{ft.} \\ 2\text{d.} \\ 3\text{d.} \\ 4\text{th.} \\ 5\text{th.} \end{array} \right\} \text{ Ordinate. } \left\{ \begin{array}{l} CB = 12. \\ CD = 16. \\ CE = 18.3303. \\ CF = 19.5959. \\ CG = 20. \end{array} \right.$$

which being substituted, in the Order they are written, for b, c, d, e , and f respectively, and 20 for x , in *Theorem* the 5th Table the 3d, there will come out 310.15343 for the Area of $\frac{1}{4}$ Part of the Circle in Inches; wherefore 310.1534×4 , or 1240.6137 divided by 282 gives 4.399 Ale-Gallons, or by 231 gives 5.37 Wine-Gallons; nearly the same, as was before determined, by the usual Method in Page 38th. And by a Method not much unlike the Precedent; the Area of any Segment, or Portion thereof, may also be determined.

Example 2. *Wherein it is proposed, to find the Area of a Conic-parabola, whose Absciss is 40, and bounding Ordinate 35 Inches.* Let the Absciss HK (FIG. 7.) be divided into 4 equal Parts, which in the present Case will be sufficient, upon account of the great Similitude, subsisting between the proposed Curve, and that in the general Proposition, and each will be 10; wherefore, by the Property of the Parabola in Page 269,

$$\text{The } \left\{ \begin{array}{l} 1\text{ft.} \\ 2\text{d.} \\ 3\text{d.} \\ 4\text{th.} \end{array} \right\} \text{ Ordinate. } \left\{ \begin{array}{l} CB = 17.5. \\ CD = 24.74815. \\ CE = 30.3107. \\ CK = 35. \end{array} \right.$$

which, together with the Absciss, 40, being substituted in *Theorem* the 4th, Table the 3d, for b, c, d, e , and x respectively, there will come out

920.85, for the Area of $\frac{1}{2}$ the Parabola in Inches; wherefore 1841.7068 divided by 282 gives 6.53 Ale-Gallons; or divided by 231 gives 7.97 Wine-Gallons: But here it may not be amiss to observe, as it has no where been taken Notice of before, that the Area of any Conic-Parabola may be determined with more Exactness and Expedition, by multiplying the Absciss and Ordinate together, and taking two third Parts of the resulting Product; the Example here made use of, was only to shew that this Method may be extended to all Kinds of Curves, and not to shew the best Method of determining the Area of this particular Curve. If greater Exactness should be desired, it is only using a greater Number of Ordinates.

Example 3. *Where let it be required, to find the Area of an Ellipsis, whose transverse Diameter is 40, and Conjugate 36 Inches.* Let the Semi-transverse, C G, (FIG. 8.) be divided into 5 equal Parts, and each will be 4; wherefore by the Property of the Ellipsis in Page 270,

$$\text{The } \left\{ \begin{array}{l} 1^{\text{st}}. \\ 2^{\text{d}}. \\ 3^{\text{d}}. \\ 4^{\text{th}}. \\ 5^{\text{th}}. \end{array} \right\} \text{ Ordinate, } \left\{ \begin{array}{l} C B = 15.8. \\ C D = 14.4. \\ C E = 16.4972. \\ C F = 17.6363. \\ C G = 18. \end{array} \right.$$

Which being substituted for b, c, d, e, f , and 20 for x , in Theorem the 5th, Table the 3d, it will produce 279.12535, for the Area of $\frac{1}{2}$ Part of the Ellipsis, in Inches; wherefore 1116.5014 being divided by 282, gives Ale, or by 231 gives Wine-Gallons. Here, it may not be amiss to observe, that the Area of any Space comprehended by any two Lines drawn at right Angles to the Axis, and the Parts of the Curve, passing thro' their terminating Points, may by this Method, be also de-

terminated, and that independent of any other Part of the Ellipsis : And the like Remark holds good, with respect to all other Curves, if so be, that a sufficient Number of Ordinates be given, or the Property of the Curve be known, whereby they may be computed. It is further to be remarked, that any Part, or Portion, of an Ellipsis, may be determined also, by Means of a Circle described upon the transverse Axis, by this known Relation between the two Curves; *As the Diameter of the circumscribing Circle, is to the conjugate Diameter of the Ellipsis; so is the Area of the Whole, or any Part of the Circle, to the Area of the Whole, or the like Part of the Ellipsis.*

Example 4. *To find the Area of an Hyperbola, whose transverse Axe is 40, conjugate 36, and Absciss 20 Inches.*

If the Absciss (FIG. 9.) be divided into 5 equal Parts, each will be 4; whence by the Property of the Curve, in Page 271,

$$\text{The } \left\{ \begin{array}{l} 1\text{ft.} \\ 2\text{d.} \\ 3\text{d.} \\ 4\text{th.} \\ 5\text{th.} \end{array} \right\} \text{ Ordinate, } \left\{ \begin{array}{l} CB = 11.94. \\ CD = 17.64. \\ CE = 22.48. \\ CF = 26.94. \\ MO = 31.18. \end{array} \right.$$

Which being substituted for b, c, d, e, f , and 20 for x , in Theorem the 5th, Table the 3d, there will from thence arise 382.9458, for the Area of half the Hyperbola, in Inches; wherefore 765.8916, divided by 282 gives 2.72 Ale, or by 231 gives 3.32 Wine Gallons.

Example 5. *Let it be required, to find the Area of an hyperbolic Space, MCB O, (see the last FIG.) comprehended by any two perpendicular Ordinates, BC = 11.94, MO = 31.18, and Part of the Axis, BO = 16, passing thro' the two extream Points, O, B,*

on

on one Hand, and Part of the Curve passing thro' the other extream Points, C, M, on the other Hand; the transverse Axe of which is 40, and the Conjugate 36?

Before one proceeds, to compute the Area of this hyperbolic Space, it will be necessary, first, to determine the Absciss TB , in Order that the equidistant Ordinates, insiting on the given Part of the Axis BO , may also be determined; and this may be easily done, by the Property of the Curve in Page 271, and will come out equal to 4 very nearly; wherefore by dividing the given Line OB into four equal Parts, and proceeding in the Manner before directed, we shall have for

$$\text{The } \left\{ \begin{array}{l} \text{2d.} \\ \text{3d.} \\ \text{4th.} \end{array} \right\} \text{ Ordinate, } \left\{ \begin{array}{l} CD = 17.64. \\ CE = 22.48. \\ CF = 26.94. \end{array} \right.$$

Which together with the first and last, that are given Ones, and the Value of x , = 16, being properly substituted in the 1st, or 2d Table, there will come out 355.228 for the Area in Inches; which being divided by 282 gives Ale, or 231 gives Wine Gallons.

From what has been already said, it is easy to infer, that this Method of determining the Areas of curvilinear Spaces, is not only accommodated to all Curves, whose Properties are known, but even to those whose Properties are unknown: And this no one can dispute, since it depends entirely upon the Ordinates themselves, and their common Distance from one another being given: Therefore, where the Property of the Curve is not known, the Ordinates, and their common Distance, must be determined by actual Dimensions, and afterwards applied in the very same Manner, as those we have been speaking of, which were deduced from the Properties of their respective Curves.

But the Usefulness of the Theorems, in the Tables, will more remarkably appear, in determining the Contents of Solids; particularly such as are now used; by many Distillers and others, commonly called elliptical Backs; and that upon Account of the Figure of their Bases; but this Appellation does not seem properly to belong to them; for they are in Fact neither Ovals, nor Ellipses; nor are they described by any certain Rule: Wherefore, we can have Recourse to no other Method, but that which is here proposed, whereby we can, with any Degree of Certainty, come at the true Contents of these Kinds of Solids. An Instance of which shall be given at the Conclusion of this Paper, from Dimensions actually taken, and this alone will sufficiently direct, how to proceed with the like, or any other, Form.

Here it may not perhaps be amiss to observe, *That all Solids have the same Relation to the Area of their respective Sections, made by Planes cutting the Axis, at right Angles, and at equal Intervals, as curvilinear Spaces have, to their respective Ordinates, insisting upon the Absciss, or Axis, at equal Intervals.*

Example 6. *In which it is proposed to find the Content of a Cone; whose Diameter at the Base is 28.8, and Altitude 30 Inches.*

If the Altitude of the Cone (FIG. 10.) be divided into any Number of equal Parts, for Instance 6; the Diameters, of the several circular Sections, passing through the Points of Division, *d, e, f, g, h*, may, by similar Triangles, thus be determined.

<i>A.C.A.S.</i>	<i>D.B.</i>	<i>Diameters.</i>	<i>Areas.</i>
30 : 5 :: 28.8 :	4.8 =	<i>DD</i>	18.09.
30 : 10 :: 28.8 :	9.6 =	<i>EE</i>	72.38.
30 : 15 :: 28.8 :	14.4 =	<i>FF</i>	162.86.
30 : 20 :: 28.8 :	19.2 =	<i>GG</i>	289.53.
30 : 25 :: 28.8 :	24.0 =	<i>HH</i>	452.39.
* * *	28.8 =	<i>CC</i>	651.44.

Whence the

Which

Which being substituted for b, c, d, e, f, g , and 30 for x , in Theorem the 6th, Table the 3d, there will come out 6437.2212, for the Content, in Inches ; wherefore 6437.2212 divided by 282 gives 22.83 Ale, or by 231 gives 27.86 Wine Gallons ; nearly the same as was before determined in Page 65: But this Operation might have been rendered much more simple, by considering, that the Diameters of any Number of circular Sections, of a Cone, made by Planes cutting the Axis, at right Angles, and at equal Intervals, are one to another, as the Terms of the Arithmetical Progression 1, 2, 3, 4, &c. and consequently the Areas, one to another, as the Squares of those Terms.

Example 7. *Where let it be proposed, to find the Content of a Cask, in the Form of the middle Frustum of a Spheroid ; whose Head Diameter is 25, Bung 31 and Length 46 Inches.*

Let GH (FIG. 11.) be any Diameter, taken between the Head and Bung, and FI the Semi-transverse of the generating Ellipsis ; then, by the Property of the Curve in Page 270, $FI^2 : CD^2 :: \overline{FI + FK} \times \overline{FI - FK} : GH^2 = CD^2 - \frac{CD^2 \times FK^2}{FI^2}$; but when FK becomes $= EF$, then will GH become $= AB$; and therefore $AB^2 = CD^2 - \frac{CD^2 \times EF^2}{FI^2}$; or $AB^2 \times FI^2 = CD^2 \times FI^2 - CD^2 \times EF^2$; consequently $FI^2 = \frac{CD^2 \times EF^2}{CD^2 - AB^2}$; which being substituted for FI^2 , in the Value of CH^2 found above, there will come out $GH^2 = CD^2 - \frac{CD^2 - AB^2 \times FK^2}{EF^2}$;

that is, as the Square of half the Length, is to the Square of that Part intercepted between the Bung, and any Diameter sought ; so is the Difference of the Squares of the Head and Bung Diameters, to a fourth Proportional ; which fourth Proportional, being sub-
tracted

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tracted from the Square of the Diameter at the Bung, leaves the Square of the Diameter sought: But if the Diameter in the Middle, between the Head and Bung be only required; then the foregoing Theorem will be reduced to $GH^2 = CD^2 + \frac{AB^2 - CD^2}{4}$
 $= \frac{AB + 3CD^2}{4}$; that is, if to the Square of the Diameter at the Head, be added three times the Square, of that at the Bung; one fourth Part of that Sum, will be the Square of the Diameter in the Middle, between the Head and Bung: Hence the Area of a Circle, having the same Diameter, as that at the Head, will be 490.88, as that in the Middle, 688.80, and as that at the Bung, 754.77 Inches: Now if 981.76 twice the first Area be substituted for A, 1377.60 twice the second for B, 754.77, the third for C, and 46 for x, in Theorem the 4th, Table the 2d, it will produce 30673.18 for the Content, in Inches; which, being divided by 282 gives 108.7 Ale, or by 231 gives 132.7 Wine Gallons; the very same as was before determined in Page 82.

Example 8. Let it be proposed, to find the Content of a Cask, in the Form of the Middle Frustum of a parabolic Spindle; whose head Diameter is 25, Bung 31, and Length 46.

Let GH (FIG. 12.) be any Diameter, taken between the Head and Bung, and CF the Absciss, of the generating Parabola; then if from the Points B, G , be drawn BL, GM parallel to FI , by the Property of the Curve, in Page 270, $CL : CM :: BL^2 : GM^2$; wherefore $2CM = \frac{2CL \times GM^2}{BL^2}$; but because $CL = \frac{CD - AB}{2}$, $GM = FK$, and $BL = EF$; it will be $2CM = \frac{CD - AB \times FK^2}{EF^2}$; which there-

therefore being taken from CD , the Diameter at the Bung, leaves $CD - \frac{CD - AB \times FK^2}{EF^2} = GH$;

that is, *as the Square of half the Length, is to the Square of that Part intercepted between the Bung and any Diameter sought; so is the Difference of the Diameters at the Head and Bung, to a fourth Proportional; which, fourth Proportional, being taken from the Diameter at the Bung, leaves the Diameter sought:* But if GH be taken in the Middle between the Diameters, at the Head and Bung; then will $\frac{AB + 3CD}{4} = GH$; that is, *if to the Diameter at*

the Head, be added three times that at the Bung, one fourth Part of that Sum, will be the Middle Diameter: Hence the Area of a Circle, having the same Diameter, as that at the Head, will be 490.88, as that in the Middle, 683.49, and as that at the Bung 754.77, Inches; wherefore if 981.76 twice the first Area be substituted for A , 1366.98 twice the second for B , 754.77 once for C , and 46 for x , in Theorem the 4th, Table the 2d, there will come out 30498.38 for the Content, in Inches; wherefore 30498.38 divided by 282 gives 108.15 Ale, or by 231 gives 132.02 Wine Gallons.

Example 9.. *Where let it be proposed, to find the Content of a Cask, in the Form, of the middle Frustum of two equal parabolic Conoids, joined together at their Bases; whose Head Diameter is 25, Bung 31, and Length 46.*

Let GH (FIG. 13.) be any Diameter taken between the Head and Bung. and FI the Absciss of the generating Parabola; then, by the Property of the Curve in Page 270, $CD^2 : AB^2 :: FI : EI$ and $CD^2 : GH^2 :: FI : IK$; whence, by Division $CD^2 - AB^2 : CD^2 :: FI - EI : FI$, and $CD^2 - GH^2 : CD^2 :: FI - IK : FI$; therefore from the former $FI = \frac{CD^2 \times FI - EI}{CD^2 - AB^2}$, and from

the

the latter $FI = \frac{CD^2 \times FI - IK}{CD^2 - GH^2}$; consequently
 $\overline{FI - EI} \times \overline{CD^2 - GH^2} = \overline{FI - IK} \times \overline{CD^2 - AB^2}$;
 but $FI - EI = EF$, and $FI - IK = FK$;
 wherefore $EF \times \overline{CD^2 - GH^2} = FK \times \overline{CD^2 - AB^2}$;
 or $EF \times \overline{CD^2 - FK \times CD^2 - AB^2} = EF \times GH^2$;
 consequently $GH^2 = CD^2 - \frac{FK \times \overline{CD^2 - AB^2}}{EF}$;

that is, as *half the Length*, is to that Part intercepted
 between the Bung and any Diameter sought; so is the Dif-
 ference of the Squares of the Diameters at Head and
 Bung, to a fourth Proportional; which, fourth Propor-
 tional, being subtracted from the Square of the Diameter
 at the Bung, leaves the Square of the Diameter sought:
 But if the Diameter in the Middle, between the
 Head and Bung be only required; then will $FK =$
 $\frac{EF}{2}$, and consequently $GH^2 = \frac{CD^2 + AB^2}{2}$; that

is, *half the Sum of the Squares of the Head and Bung
 Diameters, will in this Case, be equal to the Square of
 the middle Diameter.* Hence the Area of a Circle,
 having the same Diameter, as that at the Head,
 will be 490.88, as that in the Middle, 622.82 and
 as that at the Bung, 754.77 Inches; wherefore if
 981.76 twice the first Area, be substituted for *A*,
 1245.64 twice the second for *B*, 754.77 once the
 third for *C*, and 46 for *x*, in Theorem the 4th
 Table the 2d; there will from thence arise 28514.
 91 for the Area in Inches; which, being divided
 by 282 gives 101.12 Ale, or by 231 gives 122.
 44 Wine Gallons.

Example 10. *Let it be proposed to find the Con-
 tent of a Cask, in the Form of two equal Frustums
 of a Cone, joined together at their Bases; the Head
 Diameter of which is 25, Bung 31, and Length 46.*

Let *GH* (FIG. 14.) be a Diameter, taken between
 the Head and Bung; also let *GM*, *BL* be drawn
 perpendicular to the Diameter *CD*, at the Bung;
 then,

then, by similar Triangles, $BL : GM :: CL : CM$;
whence $2 CM = \frac{GM \times 2CL}{BL}$, which therefore taken from CD , leaves $GH = CD - \frac{GM \times 2CL}{BL}$. That is, as the Length, is to the Part intercepted between the Bung and the Diameter sought; so is the Difference of the Diameters at the Head and Bung, to a fourth Proportional; which fourth Proportional, taken from the Diameter at the Bung, leaves the Diameter sought: But if the Middle Diameter be only required; then $GM = \frac{BL}{2}$, and consequently $GH = CD - \frac{CD - AB}{2} = \frac{AB + CD}{2}$. That is, the Middle Diameter, is equal to half the Sum of the Diameters, at the Head and Bung. Hence the Area of a Circle, having the same Diameter, as that at the Head will be 490.88, as that in the Middle 615.75, and as that at the Bung 754.77; wherefore if 981.76, twice the first Area, be substituted for A , 1231.50, twice the second for B , 754.77, the third for C , and 46 for x , in Theorem the 4th, Table the 2d, there will come out 28283.64 for the Content in Inches; which being divided by 282 gives 100.29 Ale, or by 231 gives 122.44 Wine Gallons.

Example 11. Where it is proposed, to find the Content of a Solid, in the Form of one of the curvilinear Backs, or Tuns, before spoken of; the equidistant Ordinates insisting on the Axis of the several Sections, made by Planes cutting the Axis of the Solid at right Angles, and at equal Intervals, reckoning from the Top downwards, were in this Instance as follows, viz. the Ordinates $Aa, Aa, Aa, \&c.$ (FIG. 15.) insisting on the Axis $Bb = 151.7$, were 61.4, 101.2, 118.0, 123.7, 125.2, 124.0, 118.6, 99.9, 00 57.0;

57.0 ; the Ordinates Bb, Bb, Bb, &c. insisting on the Axis Cc = 153.3, were 64.7, 103.1, 119.4, 125.2, 126.3, 125.3, 119.9, 102.1, 62.2 ; the Ordinates Cc, Cc, Cc, &c. insisting on the Axis Dd = 155.1, were 67.8, 105.0, 120.9, 126.6, 127.6, 126.7, 121.4, 104.3, 66.40 ; their common Distance hi = ik = kl was 17 ; the common Distance mn = mE of the Sections BAbAB, CBcBC, DCdCD, was 20 Inches ; the Drip, or Fall 57 Ale-Gallons, and the wet Inches 1.9.

Let the Sum of the first, last, and twice the Middle Ordinates (= 368.8,) be substituted for *A* ; the Sum of the second, fourth, sixth, and eighth Ordinates (= 448.8) for *B* ; the Sum of the third, and seventh Ordinates (= 236.6) for *C*, and 68, half the Distance of the extreme Ordinates for *x*, in Theorem the 5th, Table the 2d, and it will produce 14946.7, for the Area of the curvilinear Space *AAAbAabA* ; to which adding the Areas of the two Ends *ABabA*, and *AbabA*, which are here considered as Parabolas, there will come out 15566.3, for the Area of the Section *BAbAB* : And after the very same Manner, the Area of the Sections *CBcBC*, and *DCdCD* may be determined, and will come out equal to 15926.39 and 16295.38 respectively : Hence, if the Sum of the first, and last Areas 31861.68, be added to four Times the second = 63705.56, and that Result be multiplied by one sixth Part of the Depth, as Theorem the 3d, in Table the second directs, there will come out 955672.4, for the Content in Inches ; which, being divided by 282, gives 3388.9 Ale, or by 231 gives 4137.1 Wine Gallons.

But if the Content of this Solid, be determined by the Methods commonly practised upon these Occasions ; (that is, by taking it for granted, to have elliptical Bases ; then finding the Area of the great-

er Base, the Area of the lesser, and a geometrical Mean between the two Areas, and multiplying the Sum of these three Results, by one third of the Depth; there will come out 913754.4 for the Content in Inches; which being divided by 282, gives 3240.2 Ale, and by 231 gives 3955.6 Wine Gallons. Which manifestly differs from the preceding Method 148.7 Gallons of Ale, and 181.5 Gallons of Wine: In Consequence whereof, it is evident, that Backs, or Tuns, might be so made, that the Revenue might greatly suffer by them, and that insensibly; but if this Method be duly attended to, no such Deception can ever happen hereafter.

As the foregoing Lemma, may also be applied to very good Purpose, in determining the Areas of Curves, defined by Equations, in a general Manner; it may not perhaps be altogether unacceptable, to give an Instance thereof: Let that be, in finding the Area of a Curve, whose Absciss is x , and Ordinate $dx^{pn-1} \times e + \sqrt{fx^{nm}}$: To this End, let 1, 1, 2, 3, &c. be successively written for the Index m , in the Ordinate proposed, and it will produce the following Series of equidistant Ordinates $dx^{pn-1} \times e + \sqrt{fx^{n-1}}$, $dx^{pn-1} \times e + \sqrt{fx^{n-1}}$, $dx^{pn-1} \times e + \sqrt{fx^{n-1}}$, $dx^{pn-1} \times e + \sqrt{fx^{n-1}}$, &c. among which the proposed one takes Place, when the Binomial $e + \sqrt{fx^n}$ is raised to that Power, whose Index is equal to the Number m , denoting its Distance from the first; wherefore the Area sought will also take Place, at the like Distance from the first, among the Areas answering to those Ordinates, which constitute the following Progression of equidistant

Terms, $\frac{dx^{pn}}{pn}, \frac{dex^{pn}}{pn} + \frac{dfx^{pn} + n}{pn+n}, \frac{de^2 x^{pn}}{pn} + \frac{2defx^{pn} + n}{pn+n} +$
 $\frac{df^2 x}{pn+2n}, \frac{de^3 x}{pn} + \frac{3de^2 fx}{pn+n} +$

+ $\frac{3 d e f^2 x^{pn+2n}}{pn+2n} + \frac{d f^3 x^{pn+3n}}{pn+3n}$, &c. In order to determine which Area, in as commodious a Form as need be, let each Term be divided by the Binomial $e + f x$ raised to that Power, whose Index is equal to the Number, denoting the Place of that Term from the first, and let the first Result be put $= a$, the second $= b$, the third $= c$, and so on; that is,

$$a = \frac{d x}{p n} \times \frac{e + f x^{n+1}}{e + f x^{n+1}},$$

$$b = x^{pn} \times \frac{\frac{d e}{p n} + \frac{d f x^n}{pn+n}}{e + f x^{n+1}},$$

$$c = x^{pn} \times \frac{\frac{d e^2}{p n} + \frac{2 d e f x^n}{pn+n} + \frac{d f^2 x^{2n}}{pn+2n}}{e + f x^{n+2}},$$

$$d = x^{pn} \times \frac{\frac{d e^3}{p n} + \frac{3 d e^2 f x^n}{pn+n} + \frac{3 d e f^2 x^{2n}}{pn+2n} + \frac{d f^3 x^{3n}}{pn+3n}}{e + f x^{n+3}},$$

&c.

Whence A is $= \frac{- d f x^{pn+n}}{pn \times p+1 \times e + f x^{n+1}}$, $B =$

$$\frac{2 d f^2 x^{pn+2n}}{pn \times p+1 \times p+2 \times e + f x^{n+2}}, C =$$

$$\frac{- 6 d f^3 x^{pn+3n}}{pn \times p+1 \times p+2 \times p+3 \times e + f x^{n+3}}, \text{ \&c.}$$

which together with the Value of a , being properly substituted for $a, A, B, C, \text{ \&c.}$ and m for n respectively, in the foregoing Lemma, it will produce

$$\text{duce } \frac{dx^{pn}}{pn} - \frac{dmfx^{pn+n}}{pn \times p + 1 \times e + fx^{n^1}} +$$

$$\frac{dm \times m - 1 \times f^2 x^{pn+n}}{pn \times p + 1 \times p + 2 \times e + fx^{n^2}} +$$

$$\frac{dm \times m - 1 \times m - 2 \times f^3 x^{pn+n} + 3^n}{pn \times p + 1 \times p + 2 \times p + 3 \times e + fx^{n^3}}, \text{ \&c. for the Term,}$$

whose Place from the first, is expressed by the Number m ; but because each Term was before Substitution, divided by the Binomial $e + fx^n$, raised to that Power, whose Index denoted its Place from the first; it is manifest, that if the Term, already found, be multiplied by the Power of the foresaid Binomial, whose Index is m , denoting its Place from the first, it will produce $\frac{dx^{pn}}{pn} \times$

$$e + fx^n)^m \text{ into } 1 - \frac{mf x^n}{p + 1 \times e + fx^n} +$$

$$\frac{m \times m - 1 \times f^2 x^{2n}}{p + 1 \times p + 2 \times e + fx^{n^2}} -$$

$$\frac{m \times m - 1 \times m - 2 \times f^3 x^{3n}}{p + 1 \times p + 2 \times p + 3 \times e + fx^{n^3}}, \text{ \&c, for the Area}$$

of the proposed Curve, Q. E. I.

Corollary 1. Hence, the Area of the Segment of a Circle, whose Diameter is d , versed Sine v , and Chord y , may easily be determined: For, by the Property of the Circle, $dv - v^2 = \frac{y^2}{4}$, and consequently $y = 2 \times \sqrt{dv - v^2} = 2 \times v^{\frac{1}{2}} \times d - v^{\frac{1}{2}}$; which being compared with the general Ordinate, $dx^{pn-1} \times e + fx^n)^m$, we have $m = \frac{1}{2}$, $n = 1$, $pn - 1 = \frac{1}{2}$, $\therefore p = \frac{3}{2}$, $x = v$, $e = d$, $f = -1$, and $d = 2$; consequently the Area sought, because $2y =$

$$4 \sqrt{dv - v^2}, \text{ will be } 2y \text{ into } \frac{v}{1 \times 3} +$$

$$\frac{v^2}{1 \times 3 \times 5 \times d - v} - \frac{v^3}{3 \times 5 \times 7 \times d - v^2} + \frac{v^4}{5 \times 7 \times 9 \times d - v^3}$$

$$-\frac{v^5}{7 \times 9 \times 11 \times d - v^2}, \&c.$$

Corollary 2. Hence, the Area of the Segment of an Ellipse, whose transverse Axe is t , conjugate c , Absciss x , and Ordinate y , may likewise be determined:

For, by the Property of the Ellipse $2y = \frac{2c}{t}$

$$\sqrt{tx - x^2} = \frac{2c}{t} \times x^{\frac{1}{2}} \times t - x^{\frac{1}{2}}; \text{ which being com-}$$

pared with the general Ordinate, $d x^{p^{n-1}} \times e + f x^n^m$,

we have $d = \frac{2c}{t}$, $x = x$, $t = e$, and all the rest

as in the foregoing Corollary; consequently the

Area required = $4y$ into $\frac{x}{1 \times 3} + \frac{x}{1 \times 3 \times 5 \times d - x}$

$$-\frac{x^3}{3 \times 5 \times 7 \times d - x^2} + \frac{x^4}{5 \times 7 \times 9 \times d - x^3} -$$

$$\frac{x^5}{7 \times 9 \times 11 \times d - x^4}, \&c.$$

Corollary 3. Hence, the Area of an Hyperbola whose transverse Axe is t , Conjugate c , Absciss x , and Ordinate y , may also be determined: For, by the

Property of the Hyperbola, $2y = \frac{2c}{t} \sqrt{tx + x^2} =$

$$\frac{2c}{t} \times x^{\frac{1}{2}} \times t + x^{\frac{1}{2}}; \text{ which being compared with}$$

the general Ordinate $d x^{p^{n-1}} \times e + f x^n^m$, we have $f = +1$, and the rest, as in Corollary the

second; consequently $4y$ into $\frac{x}{1 \times 3} - \frac{x^2}{1 \times 3 \times 5 \times t + x}$

$$-\frac{x^3}{3 \times 5 \times 7 \times t + x^2} - \frac{x^4}{5 \times 7 \times 9 \times t + x^3} -$$

$$\frac{x^5}{7 \times 9 \times 11 \times t + x^4} - \frac{x^6}{9 \times 11 \times 13 \times t + x^5}, \&c. \text{ will be the Area required.}$$

Example 1. Let it be required to find the Area of the Segment of a Circle, whose versed Sine is 10, and Chord,

Chord, or Subtense 40; from whence the Diameter will be 50.

Let 10 be substituted for v , 40, for y , and 50 for d , in the Series of Corollary the 1st, and it will

produce 80 into $\frac{10}{1 \times 3} + \frac{10^2}{13 \times 5 \times 40} - \frac{10^3}{3 \times 5 \times 7 \times 40^2}$

$+ \frac{10^4}{5 \times 7 \times 9 \times 40^3} - \frac{10^5}{7 \times 9 \times 11 \times 40^4} + \frac{10^6}{9 \times 11 \times 13 \times 40^5} -$

$\frac{10^7}{11 \times 13 \times 15 \times 40^6}, \&c. = \frac{800}{1 \times 3} + \frac{800}{3 \times 5 \times 4} -$

$\frac{800}{3 \times 5 \times 7 \times 4^2} + \frac{800}{5 \times 7 \times 9 \times 4^3} - \frac{800}{7 \times 9 \times 11 \times 4^4} +$

$\frac{800}{9 \times 11 \times 13 \times 4^5} - \frac{800}{11 \times 13 \times 15 \times 4^6}, \&c.$

Whence the Sum of the negative Terms, reduced to Decimals, being taken from the Sum of the affirmative ones, reduced in the same Manner, there will come out 279.55, for the Area of the Segment in Inches; which divided by 282 gives Ale, and by 231 gives Wine Gallons: But the following Approximation is much better adapted to ordinary Practice, than the foregoing general Series, and in most Instances sufficiently exact. If the versed Sine of the Segment of a Circle be v , the Chord y , and Diameter d ; then will the

Area be nearly equal to $\frac{70dv - 46v^2}{105d - 90v}$. That is, if

the Difference between seventy Times the Rectangle of the Diameter and versed Sine, and forty six Times the Square of the versed Sine; be divided by the Difference betwixt one hundred and five Times the Diameter, and ninety Times the versed Sine, and the Quotient thence arising be multiplied by the Chord, it will produce the Area in Inches; which divided by 282 gives Ale, and by 231 gives Wine Gallons. The same Approximation may also be applied to the Segment of an Ellipse,

Ellipse, if due Regard be had to the Relation between the two Curves, mentioned in Page 274; but in order to avoid Mistakes, it may be expressed in Words as follows. *From seventy Times the Rectangle of the transverse Diameter and Absciss, subtract forty six Times the Square of the Absciss; then multiply the Remainder by twice the Ordinate, and that Product by the conjugate Diameter, for a Dividend; then from one hundred and five Times the transverse Diameter, subtract ninety Times the Absciss, and multiply the Remainder by the transverse Diameter for a Divisor; then the Quotient arising from dividing the Dividend by the Divisor, will produce the Area of the Segment.* These Rules are so obvious, that they require no Examples in Numbers to illustrate them. We might indeed, here also have given Approximations for the Hyperbola, and other Curves; but it seems no ways necessary, since they seldom or ever occur in Practice, and may all be referred to the foregoing general Method.



F I N I S.



